

Rolling bearings — Explanatory notes on ISO 281

Part 1: Basic dynamic load rating and basic rating life

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National foreword

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Contents

Page

Foreword.....	iv
Introduction	v
1 Scope	1
2 Normative references	1
3 Symbols	1
4 Basic dynamic load rating	3
4.1 Basic dynamic radial load rating, C_r, for radial ball bearings	4
4.2 Basic dynamic axial load rating, C_{a1}, for single row thrust ball bearings.....	7
4.3 Basic dynamic axial load rating, C_{a2}, for thrust ball bearings with two or more rows of balls	9
4.4 Basic dynamic radial load rating, C_r, for radial roller bearings.....	10
4.5 Basic dynamic axial load rating, C_{a1}, for single row thrust roller bearings	12
4.6 Basic dynamic axial load rating, C_{a2}, for thrust roller bearings with two or more rows of rollers	13
5 Dynamic equivalent load.....	15
5.1 Expressions for dynamic equivalent load.....	15
5.2 Factors X, Y, and e.....	27
6 Basic rating life	38
7 Life adjustment factor for reliability.....	39
Bibliography	40

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

In exceptional circumstances, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example), it may decide by a simple majority vote of its participating members to publish a Technical Report. A Technical Report is entirely informative in nature and does not have to be reviewed until the data it provides are considered to be no longer valid or useful.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TR 1281-1 was prepared by Technical Committee ISO/TC 4, *Rolling bearings*, Subcommittee SC 8, *Load ratings and life*.

This first edition of ISO/TR 1281-1, together with the first edition of ISO/TR 1281-2, cancels and replaces the first edition of ISO/TR 8646:1985, which has been technically revised.

ISO/TR 1281 consists of the following parts, under the general title *Rolling bearings — Explanatory notes on ISO 281*:

- *Part 1: Basic dynamic load rating and basic rating life*
- *Part 2: Modified rating life calculation, based on a systems approach of fatigue stresses*

Introduction

ISO/R281:1962

A first discussion on an international level of the question of standardizing calculation methods for load ratings of rolling bearings took place at the 1934 conference of the International Federation of the National Standardizing Associations (ISA). When ISA held its last conference in 1939, no progress had been made. However, in its 1945 report on the state of rolling bearing standardization, the ISA 4 Secretariat included proposals for definition of concepts fundamental to load rating and life calculation standards. This report was distributed in 1949 as document ISO/TC 4 (Secretariat-1)1, and the definitions it contained are in essence those given in ISO 281:2007 for the concepts “life” and “basic dynamic load rating” (now divided into “basic dynamic radial load rating” and “basic dynamic axial load rating”).

In 1946, on the initiative of the Anti-Friction Bearing Manufacturers Association (AFBMA), New York, discussions of load rating and life calculation standards started between industries in the USA and Sweden. Chiefly on the basis of the results appearing in Reference [1], an AFBMA standard, *Method of evaluating load ratings of annular ball bearings*, was worked out and published in 1949. On the same basis, the member body for Sweden presented, in February 1950, a first proposal to ISO, “Load rating of ball bearings” [doc. ISO/TC 4/SC 1 (Sweden-1)1].

In view of the results of both further research and a modification to the AFBMA standard in 1950, as well as interest in roller bearing rating standards, in 1951, the member body for Sweden submitted a modified proposal for rating of ball bearings [doc. ISO/TC 4/SC 1 (Sweden-6)20] as well as a proposal for rating of roller bearings [doc. ISO/TC 4/SC 1 (Sweden-7)21].

Load rating and life calculation methods were then studied by ISO/TC 4, ISO/TC 4/SC 1 and ISO/TC 4/WG 3 at 11 different meetings from 1951 to 1959. Reference [2] was then of considerable use, serving as a major basis for the sections regarding roller bearing rating.

The framework for the Recommendation was settled at a TC 4/WG 3 meeting in 1956. At the time, deliberations on the draft for revision of AFBMA standards were concluded in the USA and ASA B3 approved the revised standard. It was proposed to the meeting by the USA and discussed in detail, together with the Secretariat's proposal. At the meeting, a WG 3 proposal was prepared which adopted many parts of the USA proposal.

In 1957, a Draft Proposal (document TC 4 N145) based on the WG proposal was issued. At the WG 3 meeting the next year, this Draft Proposal was investigated in detail, and at the subsequent TC 4 meeting, the adoption of TC 4 N145, with some minor amendments, was concluded. Then, Draft ISO Recommendation No. 278 (as TC 4 N188) was issued in 1959, and ISO/R281 accepted by ISO Council in 1962.

ISO 281/1:1977

In 1964, the member body for Sweden suggested that, in view of the development of imposed bearing steels, the time had come to review ISO/R281 and submitted a proposal [ISO/TC 4/WG 3 (Sweden-1)9]. However, at this time, WG 3 was not in favour of a revision.

In 1969, on the other hand, TC 4 followed a suggestion by the member body for Japan (doc. TC 4 N627) and reconstituted its WG 3, giving it the task of revising ISO/R281. The AFBMA load rating working group had at this time started revision work. The member body for the USA submitted the Draft AFBMA standard, *Load ratings and fatigue life for ball bearings* [ISO/TC 4/WG 3 (USA-1)11], for consideration in 1970 and *Load ratings and fatigue life for roller bearings* [ISO/TC 4/WG 3 (USA-3)19] in 1971.

In 1972, TC 4/WG 3 was reorganized and became TC 4/SC 8. This proposal was investigated in detail at five meetings from 1971 to 1974. The third and final Draft Proposal (doc. TC 4/SC 8 N23), with some amendments, was circulated as a Draft International Standard in 1976 and became ISO 281-1:1977.

The major part of ISO 281-1:1977 constituted a re-publication of ISO/R281, the substance of which had been only very slightly modified. However, based mainly on American investigations during the 1960s, a new clause was added, dealing with adjustment of rating life for reliability other than 90 % and for material and operating conditions.

Furthermore, supplementary background information regarding the derivation of mathematical expressions and factors given in ISO 281-1:1977 was published, first as ISO 281-2, *Explanatory notes*, in 1979; however, TC 4/SC 8 and TC 4 later decided to publish it as ISO/TR 8646:1985.

Rolling bearings — Explanatory notes on ISO 281 —

Part 1: Basic dynamic load rating and basic rating life

1 Scope

This part of ISO/TR 1281 gives supplementary background information regarding the derivation of mathematical expressions and factors given in ISO 281:2007.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 281:2007, *Rolling bearings — Dynamic load ratings and rating life*

3 Symbols

		Clause
A	constant of proportionality	7
A_1	constant of proportionality determined experimentally	4
B_1	constant of proportionality determined experimentally	4
C_1	basic dynamic radial load rating of a rotating ring	4, 5
C_2	basic dynamic radial load rating of a stationary ring	4, 5
C_a	basic dynamic axial load rating for thrust ball or roller bearing	4, 6
C_{a1}	basic dynamic axial load rating of the rotating ring of an entire thrust ball or roller bearing	4
C_{a2}	basic dynamic axial load rating of the stationary ring of an entire thrust ball or roller bearing	4
C_{ak}	basic dynamic axial load rating as a row k of an entire thrust ball or roller bearing	4
C_{a1k}	basic dynamic axial load rating as a row k of the rotating ring of thrust ball or roller bearing	4
C_{a2k}	basic dynamic axial load rating as a row k of the stationary ring of thrust ball or roller bearing	4
C_e	basic dynamic load rating for outer ring	5
C_i	basic dynamic load rating for inner ring	5
C_r	basic dynamic radial load rating for radial ball or roller bearing	4, 5, 6

D_{pw}	pitch diameter of ball or roller set	4
D_w	ball diameter	4, 5
D_{we}	mean roller diameter	4
E_o	modulus of elasticity	4
F_a	axial load	5
F_r	radial load	4, 5
J_1	factor relating mean equivalent load on a rotating ring to Q_{max}	4, 5
J_2	factor relating mean equivalent load on a stationary ring to Q_{max}	4, 5
J_a	axial load integral	5
J_r	radial load integral	4, 5
L	bearing life	7
L_{10}	basic rating life	6, 7
L_{we}	effective contact length of roller	4
L_{wek}	L_{we} per row k	4
N	number of stress applications to a point on the raceway	4
P_a	dynamic equivalent axial load for thrust bearing	5, 6
P_r	dynamic equivalent radial load for radial bearing	5, 6
P_{r1}	dynamic equivalent radial load for the rotating ring	5
P_{r2}	dynamic equivalent radial load for the stationary ring	5
Q	normal force between a rolling element and the raceways	4, 6
Q_C	rolling element load for the basic dynamic load rating of the bearing	4, 6
Q_{C1}	rolling element load for the basic dynamic load rating of a ring rotating relative to the applied load	4, 5
Q_{C2}	rolling element load for the basic dynamic load rating of a ring stationary relative to the applied load	4, 5
Q_{max}	maximum rolling element load	4, 5
S	probability of survival, reliability	4, 7
V	volume representative of the stress concentration	4
V_f	rotation factor	5
X	radial load factor for radial bearing	5
X_a	radial load factor for thrust bearing	5
Y	axial load factor for radial bearing	5
Y_a	axial load factor for thrust bearing	5
Z	number of balls or rollers per row	4, 5
Z_k	number of balls or rollers per row k	4
a	semimajor axis of the projected contact ellipse	4
a_1	life adjustment factor for reliability	7
b	semiminor axis of the projected contact ellipse	4
c	exponent determined experimentally	4, 6
c_c	compression constant	5

e	measure of life scatter, i.e. Weibull slope determined experimentally	4, 5, 6, 7
f_c	factor which depends on the geometry of the bearing components, the accuracy to which the various components are made, and the material	4
h	exponent determined experimentally	4, 6
i	number of rows of balls or rollers	4
l	circumference of the raceway	4
r	cross-sectional raceway groove radius	5
r_e	cross-sectional raceway groove radius of outer ring or housing washer	4
r_i	cross-sectional raceway groove radius of inner ring or shaft washer	4
t	auxiliary parameter	4
v	$J_2(0,5)/J_1(0,5)$	5
z_0	depth of the maximum orthogonal subsurface shear stress	4
α	nominal contact angle	4, 5
α'	actual contact angle	5
γ	$D_w \cos \alpha / D_{pw}$ for ball bearings with $\alpha \neq 90^\circ$ D_w / D_{pw} for ball bearings with $\alpha = 90^\circ$ $D_{we} \cos \alpha / D_{pw}$ for roller bearings with $\alpha \neq 90^\circ$ D_{we} / D_{pw} for roller bearings with $\alpha = 90^\circ$	4
ε	parameter indicating the width of the loaded zone in the bearing	5
η	reduction factor	4, 5
λ	reduction factor	4
μ	factor introduced by Hertz	4
ν	factor introduced by Hertz, or adjustment factor for exponent variation	4
σ_{\max}	maximum contact stress	4
$\Sigma\rho$	curvature sum	4
τ_0	maximum orthogonal subsurface shear stress	4
φ_0	one half of the loaded arc	5

4 Basic dynamic load rating

The background to basic dynamic load ratings of rolling bearings according to ISO 281 appears in References [1] and [2].

The expressions for calculation of basic dynamic load ratings of rolling bearings develop from a power correlation that can be written as follows:

$$\ln \frac{1}{S} \propto \frac{\tau_0^c N^e V}{z_0^h} \quad (1)$$

where

S is the probability of survival;

τ_0 is the maximum orthogonal subsurface shear stress;

- N is the number of stress applications to a point on the raceway;
- V is the volume representative of the stress concentration;
- z_0 is the depth of the maximum orthogonal subsurface shear stress;
- c, h are experimentally determined exponents;
- e is the measure of life scatter, i.e. the Weibull slope determined experimentally.

For “point” contact conditions (ball bearings) it is assumed that the volume, V , representative of the stress concentration in Correlation (1) is proportional to the major axis of the projected contact ellipse, $2a$, the circumference of the raceway, l , and the depth, z_0 , of the maximum orthogonal subsurface shear stress, τ_0 :

$$V \propto a z_0 l \quad (2)$$

Substituting Correlation (2) into Correlation (1):

$$\ln \frac{1}{S} \propto \frac{\tau_0^c N^e a l}{z_0^{h-1}} \quad (3)$$

“Line” contact was considered in References [1] and [2] to be approached under conditions where the major axis of the calculated Hertz contact ellipse is 1,5 times the effective roller contact length:

$$2a = 1,5L_{we} \quad (4)$$

In addition, b/a should be small enough to permit the introduction of the limit value of ab^2 as b/a approaches 0:

$$ab^2 = \frac{2}{\pi} \frac{3Q}{E_0 \sum \rho} \quad (5)$$

(for variable definitions, see 4.1).

4.1 Basic dynamic radial load rating, C_r , for radial ball bearings

From the theory of Hertz, the maximum orthogonal subsurface shear stress, τ_0 , and the depth, z_0 , can be expressed in terms of a radial load F_r , i.e. a maximum rolling element load, Q_{max} , or a maximum contact stress, σ_{max} , and dimensions for the contact area between a rolling element and the raceways. The relationships are:

$$\tau_0 = T \sigma_{max}$$

$$z_0 = \zeta b$$

$$T = \frac{(2t-1)^{1/2}}{2t(t+1)}$$

$$\zeta = \frac{1}{(t+1)(2t-1)^{1/2}}$$

$$a = \mu \left(\frac{3Q}{E_0 \sum \rho} \right)^{1/3}$$

$$b = v \left(\frac{3Q}{E_0 \Sigma \rho} \right)^{1/3}$$

where

- σ_{\max} is the maximum contact stress;
- t is the auxiliary parameter;
- a is the semimajor axis of the projected contact ellipse;
- b is the semiminor axis of the projected contact ellipse;
- Q is the normal force between a rolling element and the raceways;
- E_0 is the modulus of elasticity;
- $\Sigma \rho$ is the curvature sum;
- μ, v are factors introduced by Hertz.

Consequently, for a given rolling bearing, τ_0 , a , l and z_0 can be expressed in terms of bearing geometry, load and revolutions. Correlation (3) is changed to an equation by inserting a constant of proportionality. Inserting a specific number of revolutions (e.g. 10^6) and a specific reliability (e.g. 0,9), the equation is solved for a rolling element load for basic dynamic load rating which is designated to point contact rolling bearings introducing a constant of proportionality, A_1 :

$$Q_C = \frac{1,3}{4^{(2c+h-2)/(c-h+2)} 0,5^{3e/(c-h+2)}} A_1 \left(\frac{2r}{2r - D_w} \right)^{0,41} \frac{(1 \mp \gamma)^{(1,59c+1,41h-5,82)/(c-h+2)}}{(1 \pm \gamma)^{3e/(c-h+2)}} \times \left(\frac{\gamma}{\cos \alpha} \right)^{3/(c-h+2)} D_w^{(2c+h-5)/(c-h+2)} Z^{-3e/(c-h+2)} \quad (6)$$

where

- Q_C is the rolling element load for the basic dynamic load rating of the bearing;
- D_w is the ball diameter;
- γ is $D_w \cos \alpha / D_{pw}$;

in which

- D_{pw} is the pitch diameter of the ball set,
- α is the nominal contact angle;
- Z is the number of balls per row.

The basic dynamic radial load rating, C_1 , of a rotating ring is given by:

$$C_1 = Q_{C_1} Z \cos \alpha \frac{J_r}{J_1} = 0,407 Q_{C_1} Z \cos \alpha \quad (7)$$

The basic dynamic radial load rating, C_2 , of a stationary ring is given by:

$$C_2 = Q_{C_2} Z \cos \alpha \frac{J_r}{J_2} = 0,389 Q_{C_2} Z \cos \alpha \quad (8)$$

where

Q_{C_1} is the rolling element load for the basic dynamic load rating of a ring rotating relative to the applied load;

Q_{C_2} is the rolling element load for the basic dynamic load rating of a ring stationary relative to the applied load;

$J_r = J_r(0,5)$ is the radial load integral (see Table 3);

$J_1 = J_1(0,5)$ is the factor relating mean equivalent load on a rotating ring to Q_{\max} (see Table 3);

$J_2 = J_2(0,5)$ is the factor relating mean equivalent load on a stationary ring to Q_{\max} (see Table 3).

The relationship between C_r for an entire radial ball bearing, and C_1 and C_2 , is expressed in terms of the product law of probability as:

$$C_r = C_1 \left[1 + \left(\frac{C_1}{C_2} \right)^{(c-h+2)/3} \right]^{-3/(c-h+2)} \quad (9)$$

Substituting Equations (6), (7) and (8) into Equation (9), the basic dynamic radial load rating, C_r , for an entire ball bearing is expressed as:

$$C_r = 0,41 \frac{1,3}{4^{(2c+h-2)/(c-h+2)} 0,5^{3e/(c-h+2)}} A_1 \left[\frac{2r_i}{2r_i - D_w} \right]^{0,41} \frac{(1-\gamma)^{(1,59c + 1,41h - 5,82)/(c-h+2)}}{(1+\gamma)^{3e/(c-h+2)}} \gamma^{3/(c-h+2)} \times$$

$$\left\{ 1 + \left[1,04 \left[\frac{r_i}{r_e} \left(\frac{2r_e - D_w}{2r_i - D_w} \right) \right]^{0,41} \left(\frac{1-\gamma}{1+\gamma} \right)^{(1,59c + 1,41h + 3e - 5,82)/(c-h+2)} \right]^{(c-h+2)/3} \right\}^{-3/(c-h+2)} \times \quad (10)$$

$$(i \cos \alpha)^{(c-h-1)/(c-h+2)} Z^{(c-h-3e+2)/(c-h+2)} D_w^{(2c+h-5)/(c-h+2)}$$

where

A_1 is the experimentally determined proportionality constant;

r_i is the cross-sectional raceway groove radius of the inner ring;

r_e is the cross-sectional raceway groove radius of the outer ring;

i is the number of rows of balls.

Here, the contact angle, α , the number of rolling elements (balls), Z , and the diameter, D_w , depend on bearing design. On the other hand, the ratios of raceway groove radii, r_i and r_e , to a half-diameter of a rolling element (ball), $D_w/2$ and $\gamma = D_w \cos \alpha / D_{pw}$, are not dimensional, therefore it is convenient in practice that the value for the initial terms on the right-hand side of Equation (10) to be designated as a factor, f_c :

$$C_r = f_c (i \cos \alpha)^{(c-h-1)/(c-h+2)} Z^{(c-h-3e+2)/(c-h+2)} D_w^{(2c+h-5)/(c-h+2)} \quad (11)$$

With radial ball bearings, the faults in bearings resulting from manufacturing need to be taken into consideration, and a reduction factor, λ , is introduced to reduce the value for a basic dynamic radial load rating for radial ball bearings from its theoretical value. It is convenient to include λ in the factor, f_c . The value of λ is determined experimentally.

Consequently, the factor f_c is given by:

$$f_c = 0,41 \lambda \frac{1,3}{4^{(2c+h-2)/(c-h+2)} 0,5^{3e/(c-h+2)}} A_1 \left(\frac{2r_i}{2r_i - D_w} \right)^{0,41} \frac{(1-\gamma)^{(1,59c+1,41h-5,82)/(c-h+2)}}{(1+\gamma)^{3e/(c-h+2)}} \gamma^{3/(c-h+2)} \times \left[1 + \left\{ 1,04 \left[\frac{r_i}{r_e} \left(\frac{2r_e - D_w}{2r_i - D_w} \right) \right]^{0,41} \left(\frac{1-\gamma}{1+\gamma} \right)^{(1,59c+1,41h+3e-5,82)/(c-h+2)} \right\}^{(c-h+2)/3} \right]^{-3/(c-h+2)} \quad (12)$$

Based on References [1] and [2], the following values were assigned to the experimental constants in the load rating equations:

$$e = 10/9$$

$$c = 31/3$$

$$h = 7/3$$

Substituting the numerical values into Equation (11) gives the following, however, a sufficient number of test results are only available for small balls, i.e. up to a diameter of about 25 mm, and these show that the load rating may be taken as being proportional to $D_w^{1,8}$. In the case of larger balls, the load rating appears to increase even more slowly in relation to the ball diameter, and $D_w^{1,4}$ can be assumed where $D_w > 25,4$ mm:

$$C_r = f_c (i \cos \alpha)^{0,7} Z^{2/3} D_w^{1,8} \quad \text{for } D_w \leq 25,4 \text{ mm} \quad (13)$$

$$C_r = 3,647 f_c (i \cos \alpha)^{0,7} Z^{2/3} D_w^{1,4} \quad \text{for } D_w > 25,4 \text{ mm} \quad (14)$$

$$f_c = 0,089 A_1 0,41 \lambda \left(\frac{2r_i}{2r_i - D_w} \right)^{0,41} \frac{\gamma^{0,3} (1-\gamma)^{1,39}}{(1+\gamma)^{1/3}} \times \left[1 + \left\{ 1,04 \left(\frac{1-\gamma}{1+\gamma} \right)^{1,72} \left[\frac{r_i}{r_e} \left(\frac{2r_e - D_w}{2r_i - D_w} \right) \right]^{0,41} \right\}^{10/3} \right]^{-3/10} \quad (15)$$

Values of f_c in ISO 281:2007, Table 2, are calculated by substituting raceway groove radii and reduction factors given in Table 1 into Equation (15).

The value for $0,089A_1$ is 98,066 5 to calculate C_r in newtons.

4.2 Basic dynamic axial load rating, C_a , for single row thrust ball bearings

4.2.1 Thrust ball bearings with contact angle $\alpha \neq 90^\circ$

As in 4.1, for thrust ball bearings with contact angle $\alpha \neq 90^\circ$:

$$C_a = f_c (\cos \alpha)^{(c-h-1)/(c-h+2)} \tan \alpha Z^{(c-h-3e+2)/(c-h+2)} D_w^{(2c+h-5)/(c-h+2)} \quad (16)$$

For most thrust ball bearings, the theoretical value of a basic dynamic axial load rating has to be reduced on the basis of unequal distribution of load among the rolling elements in addition to the reduction factor, λ , which is introduced in to radial ball bearing load ratings. This reduction factor is designated as η .

Consequently, the factor f_c is given by:

$$f_c = \lambda \eta \frac{1,3}{4^{(2c+h-2)/(c-h+2)} 0,5^{3e/(c-h+2)}} A_1 \left(\frac{2r_i}{2r_i - D_w} \right)^{0,41} \frac{(1-\gamma)^{(1,59c+1,41h-5,82)/(c-h+2)}}{(1+\gamma)^{3e/(c-h+2)}} \gamma^{3/(c-h+2)} \times \left[1 + \left\{ \left[\frac{r_i}{r_e} \left(\frac{2r_e - D_w}{2r_i - D_w} \right) \right]^{0,41} \left(\frac{1-\gamma}{1+\gamma} \right)^{(1,59c+1,41h+3e-5,82)/(c-h+2)} \right\}^{(c-h+2)/3} \right]^{-3/(c-h+2)} \quad (17)$$

Similarly, to take the effect of ball size into account, substitute experimental constants $e = 10/9$, $c = 31/3$, and $h = 7/3$ into Equations (16) and (17) to give:

$$C_a = f_c (\cos \alpha)^{0,7} \tan \alpha Z^{2/3} D_w^{1,8} \quad \text{for } D_w \leq 25,4 \text{ mm} \quad (18)$$

$$C_a = 3,647 f_c (\cos \alpha)^{0,7} \tan \alpha Z^{2/3} D_w^{1,4} \quad \text{for } D_w > 25,4 \text{ mm} \quad (19)$$

$$f_c = 0,089 A_1 \lambda \eta \left(\frac{2r_i}{2r_i - D_w} \right)^{0,41} \frac{\gamma^{0,3} (1-\gamma)^{1,39}}{(1+\gamma)^{1/3}} \times \left[1 + \left\{ \left[\frac{r_i}{r_e} \left(\frac{2r_e - D_w}{2r_i - D_w} \right) \right]^{0,41} \left(\frac{1-\gamma}{1+\gamma} \right)^{1,72} \right\}^{10/3} \right]^{-3/10} \quad (20)$$

The value for $0,089A_1$ is 98,066 5 to calculate C_a in newtons. Values of f_c in ISO 281:2007, Table 4, rightmost column, are calculated by substituting raceway groove radii and reduction factors given in Table 1 into Equation (20).

4.2.2 Thrust ball bearings with contact angle $\alpha = 90^\circ$

As in 4.1, for thrust ball bearings with contact angle $\alpha = 90^\circ$:

$$C_a = f_c Z^{(c-h-3e+2)/(c-h+2)} D_w^{(2c+h-5)/(c-h+2)} \quad (21)$$

$$f_c = \lambda \eta \frac{1,3}{4^{(2c+h-2)/(c-h+2)} 0,5^{3e/(c-h+2)}} A_1 \left(\frac{2r_i}{2r_i - D_w} \right)^{0,41} \gamma^{3/(c-h+2)} \times \left[1 + \left\{ \left[\frac{r_i}{r_e} \left(\frac{2r_e - D_w}{2r_i - D_w} \right) \right]^{0,41} \right\}^{(c-h+2)/3} \right]^{-3/(c-h+2)} \quad (22)$$

in which $\gamma = D_w/D_{pw}$.

Similarly, to take the effect of ball size into account, substitute experimental constants $e = 10/9$, $c = 31/3$, and $h = 7/3$ into Equations (21) and (22), to give:

$$C_a = f_c Z^{2/3} D_w^{1,8} \quad \text{for } D_w \leq 25,4 \text{ mm} \quad (23)$$

$$C_a = 3,647 f_c Z^{2/3} D_w^{1,4} \quad \text{for } D_w > 25,4 \text{ mm} \quad (24)$$

$$f_c = 0,089 A_1 \lambda \eta \left(\frac{2r_i}{2r_i - D_w} \right)^{0,41} \gamma^{0,3} \left[1 + \left\{ \left[\frac{r_i}{r_e} \left(\frac{2r_e - D_w}{2r_i - D_w} \right) \right]^{0,41} \right\}^{10/3} \right]^{-3/10} \quad (25)$$

The value for $0,089A_1$ is 98,066 5 to calculate C_a in newtons. Values of f_c in ISO 281:2007, Table 4, second column from left, are calculated by substituting raceway groove radii and reduction factors which are given in Table 1 into Equation (25).

4.3 Basic dynamic axial load rating, C_a , for thrust ball bearings with two or more rows of balls

According to the product law of probability, relationships between the basic axial load rating of an entire thrust ball bearing and of both the rotating and stationary rings are given as:

$$C_{ak} = \left[C_{a1k}^{-(c-h+2)/3} + C_{a2k}^{-(c-h+2)/3} \right]^{-3/(c-h+2)} \quad (26)$$

$$\left. \begin{aligned} C_{a1k} &= Q_{C_1} \sin \alpha Z_k \\ C_{a2k} &= Q_{C_2} \sin \alpha Z_k \end{aligned} \right\} \quad (27)$$

$$C_a = \left[C_{a1}^{-(c-h+2)/3} + C_{a2}^{-(c-h+2)/3} \right]^{-3/(c-h+2)} \quad (28)$$

$$\left. \begin{aligned} C_{a1} &= Q_{C_1} \sin \alpha \sum_{k=1}^n Z_k \\ C_{a2} &= Q_{C_2} \sin \alpha \sum_{k=1}^n Z_k \end{aligned} \right\} \quad (29)$$

where

C_{ak} is the basic dynamic axial load rating as a row k of an entire thrust ball bearing;

C_{a1k} is the basic dynamic axial load rating as a row k of the rotating ring of an entire thrust ball bearing;

C_{a2k} is the basic dynamic axial load rating as a row k of the stationary ring of an entire thrust ball bearing;

C_a is the basic dynamic axial load rating of an entire thrust ball bearing;

C_{a1} is the basic dynamic axial load rating of the rotating ring of an entire thrust ball bearing;

C_{a2} is the basic dynamic axial load rating of the stationary ring of an entire thrust ball bearing;

Z_k is the number of balls per row k .

Substituting Equations (26), (27), and (29) into Equation (28), and rearranging, gives:

$$\begin{aligned}
 C_a &= \sum_{k=1}^n Z_k \left[\frac{\left(Q_{C_1} \sin \alpha \sum_{k=1}^n Z_k \right)^{-(c-h+2)/3} + \left(Q_{C_2} \sin \alpha \sum_{k=1}^n Z_k \right)^{-(c-h+2)/3}}{\left(\sum_{k=1}^n Z_k \right)^{-(c-h+2)/3}} \right]^{-3/(c-h+2)} \\
 &= \sum_{k=1}^n Z_k \left[\frac{\left\{ \left[\left(Q_{C_1} \sin \alpha Z_k \right)^{-(c-h+2)/3} + \left(Q_{C_2} \sin \alpha Z_k \right)^{-(c-h+2)/3} \right]^{-3/(c-h+2)} \right\}^{-(c-h+2)/3}}{Z_k^{-(c-h+2)/3}} \right]^{-3/(c-h+2)} \\
 &= \sum_{k=1}^n Z_k \left[\sum_{k=1}^n \left(\frac{Z_k}{C_{ak}} \right)^{(c-h+2)/3} \right]^{-3/(c-h+2)}
 \end{aligned}$$

Substituting experimental constants $c = 31/3$ and $h = 7/3$ gives:

$$C_a = (Z_1 + Z_2 + Z_3 + \dots + Z_n) \left[\left(\frac{Z_1}{C_{a1}} \right)^{10/3} + \left(\frac{Z_2}{C_{a2}} \right)^{10/3} + \left(\frac{Z_3}{C_{a3}} \right)^{10/3} + \dots + \left(\frac{Z_n}{C_{an}} \right)^{10/3} \right]^{-3/10} \quad (30)$$

The load ratings $C_{a1}, C_{a2}, C_{a3} \dots C_{an}$ for the rows with $Z_1, Z_2, Z_3 \dots Z_n$ balls are calculated from the appropriate single row thrust ball bearing equation in 4.2.

4.4 Basic dynamic radial load rating, C_r , for radial roller bearings

By a procedure similar to that used to obtain Equation (10) for point contact in 4.1, but applying Equations (4) and (5), the basic dynamic radial load rating of radial roller bearings (line contact) is obtained:

$$\begin{aligned}
 C_r &= 0,377 \frac{1}{2^{(c+h-1)/(c-h+1)} 0,5^{2e/(c-h+1)}} B_1 \frac{(1-\gamma)^{(c+h-3)/(c-h+1)}}{(1+\gamma)^{2e/(c-h+1)}} \gamma^{2/(c-h+1)} \times \\
 &\left\{ 1 + \left[1,04 \left(\frac{1-\gamma}{1+\gamma} \right)^{(c+h+2e-3)/(c-h+1)} \right]^{(c-h+1)/2} \right\}^{-2/(c-h+1)} (i L_{we} \cos \alpha)^{(c-h+1)/(c-h+1)} \times \\
 &Z^{(c-h-2e+1)/(c-h+1)} D_{we}^{(c+h-3)/(c-h+1)}
 \end{aligned} \quad (31)$$

where

B_1 is an experimentally determined proportionality constant;

γ is

$$D_{we} \cos \alpha / D_{pw}$$

in which D_{pw} is the pitch diameter of roller set;

D_{we} is the mean roller diameter;

α is the nominal contact angle;

L_{we} is the effective contact length of roller;

i is the number of rows of rollers;

Z is the number of rollers per row.

Here, the contact angle, α , the number of rollers, Z , the mean diameter, D_{we} , and the effective contact length, L_{we} , depend on bearing design. On the other hand, $\gamma = D_{we} \cos \alpha / D_{pw}$ is not dimensional, therefore it is convenient in practice that the terms up to “ $i L_{we} \dots$ ” on the right-hand side of Equation (31) to be designated as a factor, f_c .

Consequently,

$$C_r = f_c (i L_{we} \cos \alpha)^{(c-h-1)/(c-h+1)} Z^{(c-h-2e+1)/(c-h+1)} D_{we}^{(c-h-3)/(c-h+1)} \quad (32)$$

For the basic dynamic radial load rating for radial roller bearings, adjustments are made to take account of stress concentration (e.g. edge loading) and of the use of a constant instead of a varying life formula exponent (see Clause 6). Adjustment for stress concentration is a reduction factor, λ , and for exponent variation a factor, ν . It is convenient to include both factors — which are determined experimentally — in the factor, f_c , which is consequently given by:

$$f_c = 0,377 \lambda \nu \frac{1}{2^{(c+h-1)/(c-h+1)} 0,5^{2e/(c-h+1)} B_1 \frac{(1-\gamma)^{(c+h-3)/(c-h+1)}}{(1+\gamma)^{2e/(c-h+1)}} \gamma^{2/(c-h+1)} \times \left\{ 1 + \left[1,04 \left(\frac{1-\gamma}{1+\gamma} \right)^{(c+h+2e-3)/(c-h+1)} \right]^{(c-h+1)/2} \right\}^{-2/(c-h+1)} \quad (33)$$

The Weibull slope, e , and the constants, c and h , are determined experimentally. Based on References [1] and [2] and subsequent verification tests with spherical, cylindrical, and tapered roller bearings, the following values were assigned to the experimental constants in the rating equations:

$$e = \frac{9}{8}$$

$$c = \frac{31}{3}$$

$$h = \frac{7}{3}$$

Substituting experimental constants $e = 9/8$, $c = 31/3$, and $h = 7/3$ into Equations (32) and (33),

$$C_r = f_c (i L_{we} \cos \alpha)^{7/9} Z^{3/4} D_{we}^{29/27} \quad (34)$$

$$f_c = 0,483 B_1 0,377 \lambda \nu \frac{\gamma^{2/9} (1-\gamma)^{29/27}}{(1+\gamma)^{1/4}} \left\{ 1 + \left[1,04 \left(\frac{1-\gamma}{1+\gamma} \right)^{143/108} \right]^{9/2} \right\}^{-2/9} \quad (35)$$

The value for $0,483 B_1$ is 551,133 73 to calculate C_r in newtons. Values of f_c in ISO 281:2007, Table 7, are calculated by substituting the reduction factor given in Table 2 into Equation (35).

4.5 Basic dynamic axial load rating, C_a , for single row thrust roller bearings

4.5.1 Thrust roller bearings with contact $\alpha \neq 90^\circ$

Extension of 4.1 gives:

$$C_a = f_c (L_{we} \cos \alpha)^{(c-h-1)/(c-h+1)} \tan \alpha Z^{(c-h-2e+1)/(c-h+1)} D_{we}^{(c+h-3)/(c-h+1)} \quad (36)$$

For thrust roller bearings, the theoretical value of a basic dynamic axial load rating has to be reduced on the basis of unequal distribution of load among the rolling elements in addition to the reduction factor, λ , which is introduced in radial roller bearing load ratings. This reduction factor is designated as η .

Consequently, the factor f_c is given by:

$$f_c = \lambda \nu \eta \frac{1}{2^{(c+h-1)/(c-h+1)} 0,5^{2e/(c-h+1)}} B_1 \frac{(1-\gamma)^{(c+h-3)/(c-h+1)}}{(1+\gamma)^{2e/(c-h+1)}} \gamma^{2/(c-h+1)} \times \left\{ 1 + \left[\left(\frac{1-\gamma}{1+\gamma} \right)^{(c+h+2e-3)/(c-h+1)} \right]^{(c-h+1)/2} \right\}^{-2/(c-h+1)} \quad (37)$$

Substituting experimental constants $e = 9/8$, $c = 31/3$, and $h = 7/3$,

$$C_a = f_c (L_{we} \cos \alpha)^{7/9} \tan \alpha Z^{3/4} D_{we}^{29/27} \quad (38)$$

$$f_c = 0,483 B_1 \lambda \nu \eta \frac{\gamma^{2/9} (1-\gamma)^{29/27}}{(1+\gamma)^{1/4}} \left\{ 1 + \left[\left(\frac{1-\gamma}{1+\gamma} \right)^{143/108} \right]^{9/2} \right\}^{-2/9} \quad (39)$$

The value for $0,483 B_1$ is 551,133 73 to calculate C_a in newtons. Values for f_c in ISO 281:2007, Table 10, second column from left, are calculated by substituting reduction factors given in Table 2 into Equation (39).

4.5.2 Thrust roller bearings with contact angle $\alpha = 90^\circ$

Extension of 4.1 gives:

$$C_a = f_c L_{we}^{(c-h-1)/(c-h+1)} Z^{(c-h-2e+1)/(c-h+1)} D_{we}^{(c+h-3)/(c-h+1)} \quad (40)$$

$$f_c = \lambda \nu \eta \frac{1}{2^{(c+h-1)/(c-h+1)} 0,5^{2e/(c-h+1)}} B_1 \gamma^{2/(c-h+1)} 2^{-2/(c-h+1)} \quad (41)$$

Substituting experimental constants $e = 9/8$, $c = 31/3$ and $h = 7/3$,

$$C_a = f_c L_{we}^{7/9} Z^{3/4} D_{we}^{29/27} \quad (42)$$

$$f_c = 0,41 B_1 \lambda \nu \eta \gamma^{2/9} \quad (43)$$

The value for $0,41 B_1$ is 472,453 88 to calculate C_a in newtons. Values of f_c in ISO 281:2007, Table 10, second column from left, are calculated by substituting reduction factors given in Table 2 into Equation (43).

4.6 Basic dynamic axial load rating, C_a , for thrust roller bearings with two or more rows of rollers

According to the product law of probability, relationships between the basic dynamic axial load rating of an entire thrust roller bearing and of both the rotating and stationary rings are given as follows:

$$C_{ak} = \left[C_{a1k}^{-(c-h+1)/2} + C_{a2k}^{-(c-h+1)/2} \right]^{-2/(c-h+1)} \quad (44)$$

$$\left. \begin{aligned} C_{a1k} &= Q_{C_1} \sin \alpha Z_k L_{wek} \\ C_{a2k} &= Q_{C_2} \sin \alpha Z_k L_{wek} \end{aligned} \right\} \quad (45)$$

$$C_a = \left[C_{a1}^{-(c-h+1)/2} + C_{a2}^{-(c-h+1)/2} \right]^{-2/(c-h+1)} \quad (46)$$

$$\left. \begin{aligned} C_{a1} &= Q_{C_1} \sin \alpha \sum_{k=1}^n Z_k L_{wek} \\ C_{a2} &= Q_{C_2} \sin \alpha \sum_{k=1}^n Z_k L_{wek} \end{aligned} \right\} \quad (47)$$

C_{ak} is the basic dynamic axial load rating as a row k of an entire thrust roller bearing;

C_{a1k} is the basic dynamic axial load rating as a row k of the rotating ring of an entire thrust roller bearing;

C_{a2k} is the basic dynamic axial load rating as a row k of the stationary ring of an entire thrust roller bearing;

C_a is the basic dynamic axial load rating of an entire thrust roller bearing;

C_{a1} is the basic dynamic axial load rating of the rotating ring of an entire thrust roller bearing;

C_{a2} is the basic dynamic axial load rating of the stationary ring of an entire thrust roller bearing;

Z_k is the number of rollers per row k .

Substituting Equations (44), (45), and (47) into Equation (46), and rearranging, gives:

$$\begin{aligned}
 C_a &= \sum_{k=1}^n Z_k L_{wek} \left[\frac{\left(Q_{C_1} \sin \alpha \sum_{k=1}^n Z_k L_{wek} \right)^{-(c-h+1)/2} + \left(Q_{C_2} \sin \alpha \sum_{k=1}^n Z_k L_{wek} \right)^{-(c-h+1)/2}}{\left(\sum_{k=1}^n Z_k L_{wek} \right)^{-(c-h+2)/3}} \right]^{-2/(c-h+1)} \\
 &= \sum_{k=1}^n Z_k L_{wek} \times \\
 &\quad \left[\frac{\sum_{k=1}^n \left\{ \left[\left(Q_{C_1} \sin \alpha Z_k L_{wek} \right)^{-(c-h+1)/2} + \left(Q_{C_2} \sin \alpha Z_k L_{wek} \right)^{-(c-h+1)/2} \right]^{-2/(c-h+1)} \right\}^{-2/(c-h+1)}}{Z_k L_{wek}^{-(c-h+1)/2}} \right]^{-2/(c-h+1)} \\
 &= \sum_{k=1}^n Z_k L_{wek} \left[\sum_{k=1}^n \left(\frac{Z_k L_{wek}}{C_{ak}} \right)^{(c-h+1)/2} \right]^{-2/(c-h+1)}
 \end{aligned}$$

Substituting experimental constants $c = 31/3$ and $h = 7/3$,

$$\begin{aligned}
 C_a &= (Z_1 L_{we1} + Z_2 L_{we2} + Z_3 L_{we3} + \dots + Z_n L_{wen}) \times \\
 &\quad \left[\left(\frac{Z_1 L_{we1}}{C_{a1}} \right)^{9/2} + \left(\frac{Z_2 L_{we2}}{C_{a2}} \right)^{9/2} + \left(\frac{Z_3 L_{we3}}{C_{a3}} \right)^{9/2} + \dots + \left(\frac{Z_n L_{wen}}{C_{an}} \right)^{9/2} \right]^{-2/9} \quad (48)
 \end{aligned}$$

The load ratings, C_{a1} , C_{a2} , C_{a3} ... C_{an} for the rows with Z_1 , Z_2 , Z_3 ... Z_n rollers of lengths L_{we1} , L_{we2} , L_{we3} ... L_{wen} , are calculated from the appropriate single row thrust roller bearing equation in 4.2.

Table 1 — Raceway groove radius and reduction factor for ball bearings

Table No. in ISO 281:2007	Bearing type	Raceway groove radius		Reduction factor	
		r_i	r_e	λ	η
2	Single row radial contact groove ball bearings Single and double row angular contact groove ball bearings	0,52 D_w		0,95	—
	Double row radial contact groove ball bearings	0,52 D_w		0,90	—
	Single and double row self-aligning ball bearings	0,53 D_w	$0,5 \left(\frac{1}{\gamma} + 1 \right) D_w$	1	—
	Single row radial contact separable ball bearings (magneto bearings)	0,52 D_w	∞	0,95	—
4	Thrust ball bearings	0,535 D_w		0,90	$1 - \frac{\sin \alpha}{3}$

NOTE Values of f_c in ISO 281:2007, Tables 2 and 4, are calculated by substituting raceway groove radii and reduction factors in this table into Equations (15), (20), and (25), respectively.

Table 2 — Reduction factor for roller bearings

Table No. in ISO 281:2007	Bearing type	Reduction factor	
		λ_v	η
7	Radial roller bearings	0,83	—
10	Thrust roller bearings	0,73	$1 - 0,15 \sin \alpha$

NOTE Values of f_c in ISO 281:2007, Tables 7 and 10, are calculated by substituting reduction factors in this table into Equations (35), (39), and (51), respectively.

5 Dynamic equivalent load

5.1 Expressions for dynamic equivalent load

5.1.1 Theoretical dynamic equivalent radial load, P_r , for single row radial bearings

If the indices 1 and 2 are assigned to the ring which rotates relative to the direction of load and the stationary ring respectively, then the mean values of the rolling element loads which are decisive for a single row radial bearing ring's life are given by:

$$\left. \begin{aligned} Q_{C_1} &= Q_{\max} J_1 = \frac{F_r}{Z \cos \alpha} \frac{J_1}{J_r} = \frac{F_a}{Z \sin \alpha} \frac{J_1}{J_a} \\ Q_{C_2} &= Q_{\max} J_2 = \frac{F_r}{Z \cos \alpha} \frac{J_2}{J_r} = \frac{F_a}{Z \sin \alpha} \frac{J_2}{J_a} \end{aligned} \right\} \quad (49)$$

where

Q_{\max} is the maximum rolling element load;

J_1 is the factor relating Q_{C_1} to Q_{\max} ;

J_2 is the factor relating Q_{C_2} to Q_{\max} ;

F_r is the radial load;

F_a is the axial load;

J_r is the radial load integral;

J_a is the axial load integral;

Z is the number of rolling elements;

α is the nominal contact angle.

Radial and axial load integrals are given by:

$$\left. \begin{aligned} J_r &= J_r(\varepsilon) = \frac{1}{2\pi} \int_{-\varphi_0}^{+\varphi_0} \left[1 - \frac{1}{2\varepsilon} (1 - \cos \varphi) \right]^t \cos \varphi \, d\varphi \\ J_a &= J_a(\varepsilon) = \frac{1}{2\pi} \int_{-\varphi_0}^{+\varphi_0} \left[1 - \frac{1}{2\varepsilon} (1 - \cos \varphi) \right]^t \, d\varphi \end{aligned} \right\} \quad (50)$$

where

- t is 3/2 for point contact;
- t is 1,1 for line contact;
- φ_0 is one half of the loaded arc;
- ε is a parameter indicating the width of the loaded zone in the bearing.

Introducing the notation

$$J(t; s) = \left\{ \frac{1}{2\pi} \int_{-\varphi_0}^{+\varphi_0} \left[1 - \frac{1}{2\varepsilon} (1 - \cos \varphi) \right]^t d\varphi \right\}^{1/s} \quad (51)$$

$$\left. \begin{aligned} J_1 = J_1(\varepsilon) = J\left(\frac{9}{2}; 3\right); J_2 = J_2(\varepsilon) = J\left(5; \frac{10}{3}\right) \\ J_1 = J_1(\varepsilon) = J\left(\frac{9}{2}; 4\right); J_2 = J_2(\varepsilon) = J\left(5; \frac{9}{2}\right) \end{aligned} \right\} \quad (52)$$

for point and line contact respectively.

If P_{r1} and P_{r2} are the dynamic equivalent radial loads for the respective rings, then with radial displacement of the rings ($\varepsilon = 0,5$)

$$Q_{C1} = \frac{P_{r1}}{Z \cos \alpha} \frac{J_1(0,5)}{J_r(0,5)}; Q_{C2} = \frac{P_{r2}}{Z \cos \alpha} \frac{J_2(0,5)}{J_r(0,5)} \quad (53)$$

where the values $J_1(0,5)$, $J_2(0,5)$ and $J_r(0,5)$ are given in Table 3.

From Equations (49), (53) and

$$\left(\frac{P_r}{C_r} \right)^w = \left(\frac{P_{r1}}{C_1} \right)^w + \left(\frac{P_{r2}}{C_2} \right)^w$$

is obtained

$$\left. \begin{aligned} \frac{P_r}{F_r} &= \left[\left(\frac{C_r}{C_1} \frac{J_r(0,5)}{J_1(0,5)} \frac{J_1}{J_r} \right)^w + \left(\frac{C_r}{C_2} \frac{J_r(0,5)}{J_2(0,5)} \frac{J_2}{J_r} \right)^w \right]^{1/w} \\ \frac{P_r}{F_a \cot \alpha} &= \left[\left(\frac{C_r}{C_1} \frac{J_1}{J_1(0,5)} \right)^w + \left(\frac{C_r}{C_2} \frac{J_2}{J_2(0,5)} \right)^w \right]^{1/w} \frac{J_r(0,5)}{J_a} \end{aligned} \right\} \quad (54)$$

where

- C_r is the basic dynamic radial load rating;
- C_1 is the basic dynamic radial load rating of a rotating ring;
- C_2 is the basic dynamic radial load rating of a stationary ring;
- w is equal to pe , where p is the exponent on life formula and e is the Weibull slope.

Table 3 — Values of $J_r(0,5)$, $J_a(0,5)$, $J_1(0,5)$, $J_2(0,5)$ and w

Quantity	Point contact		Line contact		Point and line contact	
	Single row bearing	Double row bearing	Single row bearing	Double row bearing	Single row bearing	Double row bearing
$J_r(0,5)$	0,228 8	0,457 7	0,245 3	0,490 6	0,236 9	0,473 9
$J_a(0,5)$	0,278 2	0	0,309 0	0	0,293 2	0
$J_1(0,5)$	0,562 5	0,692 5	0,649 5	0,757 7	0,604 4	0,724 4
$J_2(0,5)$	0,587 5	0,723 3	0,674 4	0,786 7	0,629 5	0,754 3
$J_r(0,5)/J_a(0,5)$	0,822	—	0,794	—	0,808	—
$J_r(0,5)/J_1(0,5)$	0,407	0,661	0,378	0,648	0,392	0,654
$J_r(0,5)/J_2(0,5)$	0,389	0,633	0,364	0,623	0,376	0,628
$J_2(0,5)/J_1(0,5)$	1,044		1,038		1,041	
$\frac{J_r(0,5)}{\sqrt{J_1(0,5) J_2(0,5)}}$	0,398 ($\approx 0,40$)	0,647 ($\approx 0,65$)	0,371	0,635	0,384	0,641
w	$\frac{10}{3}$		$\frac{9}{2}$		$\frac{180}{47}$	
$2^{1-(1/w)}$	1,625		1,714		1,669	

For radial displacement of the bearing rings ($\varepsilon = 0,5$) and fixed outer ring load ($C_1 = C_i$, basic dynamic load rating for inner ring; $C_2 = C_e$, basic dynamic load rating for outer ring) from Equation (54) is found

$$\left. \begin{aligned} P_r = F_r = \frac{J_r(0,5)}{J_a(0,5)} F_a \cot \alpha = 0,822 F_a \cot \alpha \\ P_r = F_r = \frac{J_r(0,5)}{J_a(0,5)} F_a \cot \alpha = 0,794 F_a \cot \alpha \end{aligned} \right\} \quad (55)$$

for point and line contact respectively.

For $\varepsilon = 0,5$ and fixed inner ring load ($C_1 = V_f C_e$; $C_2 = C_i/V_f$), is found

$$P_r = V_f F_r \quad (56)$$

where V_f is the rotation factor.

The factor V_f varies between $1 \pm 0,044$ and $1 \pm 0,038$ for point and line contact respectively. In ISO 281:2007, the rotation factor V_f has been deleted.

NOTE The value of 1,2 for the rotation factor V_f was given in ISO/R281 for radial bearings, except self-aligning ball bearings, as safety factor.

For axial displacement of the bearing rings ($\varepsilon = \infty$) and fixed outer ring load ($C_1 = C_i$, $C_2 = C_e$),

$$\left. \begin{aligned} P_r = Y F_a \\ Y = f_1 \frac{C_i}{C_e} \frac{J_r(0,5)}{J_1(0,5)} \cot \alpha \end{aligned} \right\} \quad (57)$$

The factor $f_1(C_1/C_2)$ varies between 1 and $1/V_f = J_1(0,5)/J_2(0,5)$. Introducing as a good approximation the geometric mean value $1/\sqrt{V_f}$ between these two values (see Table 3),

$$Y = \frac{J_r(0,5)}{\sqrt{J_1(0,5) J_2(0,5)}} \cot \alpha \quad (58)$$

For non-self-aligning bearings, consideration has to be given to the effect of the manufacturing precision on the factor, Y .

The value of Y given in Equation (58) is corrected by the reduction factor η .

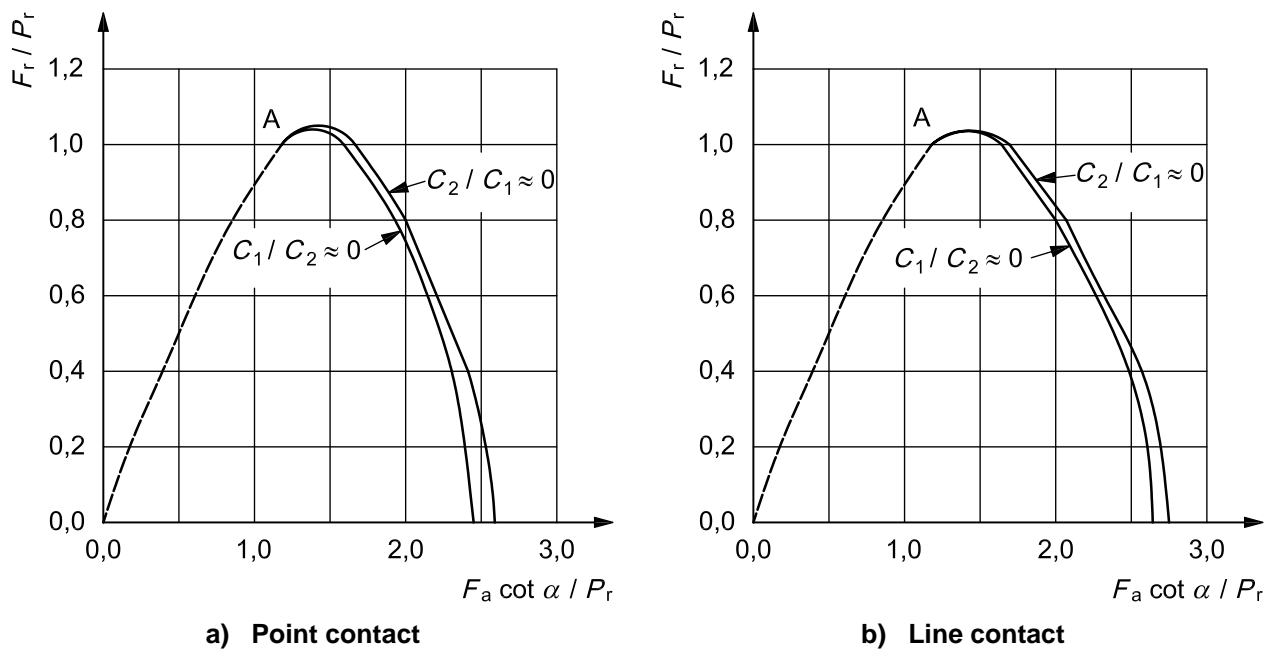
$$Y_1 = \frac{Y}{\eta} \quad (59)$$

For combined loads, Equation (54) gives related values of F_r/P_r and $F_r \cot \alpha/P_r$ corresponding to the curves given in Figure 1 for the limiting cases $C_1/C_2 \approx 0$ and $C_2/C_1 \approx 0$.

The points A represent $\varepsilon = 0,5$, i.e. radial displacement of the bearing rings. For these points,

$$\left. \begin{aligned} F_a &= 1,22 F_r \tan \alpha \\ F_a &= 1,26 F_r \tan \alpha \end{aligned} \right\} \quad (60)$$

for point and line contact, respectively.



Key

- A points A
- C_1 basic dynamic radial load rating of a rotating ring
- C_2 basic dynamic radial load rating of a stationary ring
- F_a axial load
- F_r radial load
- P_r dynamic equivalent radial load for radial bearing
- α nominal contact angle

Figure 1 — Dynamic equivalent radial load, P_r , for single row radial bearings with constant contact angle, α

5.1.2 Theoretical dynamic equivalent radial load, P_r , for double row radial bearings

For double row radial bearings, the indices I and II are assigned to the respective rows. The determining factors for life of the rotating and stationary rings are the mean values

$$\left. \begin{aligned} Q_{C_1} &= J_1 Q_{\max I} \\ Q_{C_2} &= J_2 Q_{\max II} \end{aligned} \right\} \quad (61)$$

where

$$\left. \begin{aligned} J_1 &= \left[J_1 (\varepsilon_I)^w + \left(\frac{Q_{\max II}}{Q_{\max I}} \right)^w J_1 (\varepsilon_{II})^w \right]^{1/w} \\ J_2 &= \left[J_2 (\varepsilon_I)^w + \left(\frac{Q_{\max II}}{Q_{\max I}} \right)^w J_2 (\varepsilon_{II})^w \right]^{1/w} \end{aligned} \right\} \quad (62)$$

For a bearing without internal clearances,

$$\left. \begin{aligned} \varepsilon_I + \varepsilon_{II} &= 1 & \text{for } \varepsilon_I \leq 1 \\ \varepsilon_{II} &= 0 & \text{for } \varepsilon_I > 1 \end{aligned} \right\} \quad (63)$$

If the values of J_r , J_a , J_1 and J_2 for double row bearings are introduced, then the equivalent bearing load is obtained from Equation (54), as for single row bearings. $J_r(0,5)$, $J_a(0,5)$, $J_1(0,5)$ are here the values valid for $\varepsilon_I = \varepsilon_{II} = 0,5$ (see Table 3).

The bent curves given in Figure 2 are found for the limiting cases $C_1/C_2 \approx 0$ and $C_2/C_1 \approx 0$.

Both rows are loaded if $\varepsilon_I < 1$, i.e. if

$$\left. \begin{aligned} F_a &< 1,67 F_r \tan \alpha \\ F_a &< 1,91 F_r \tan \alpha \end{aligned} \right\} \quad (64)$$

for point and line contact, respectively.

Only one row is loaded if F_a is greater than that value. In that case, the life for double row bearings can be calculated from the theory of single row bearings as well as from the theory of double row bearings.

If P_{rI} is the equivalent radial load for the loaded row considered as a single row bearing and P_r is the equivalent load for the double row bearing,

$$\frac{P_r}{P_{rI}} = \frac{C_r}{C_1} = 2^{1-(1/w)} \quad (65)$$

Figures 1 and 2 are calculated on the assumption of a constant contact angle. Figures 1 a) and 2 a) are also approximately applicable to angular contact groove ball bearings, if $\cot \alpha'$ is determined from Equation (66):

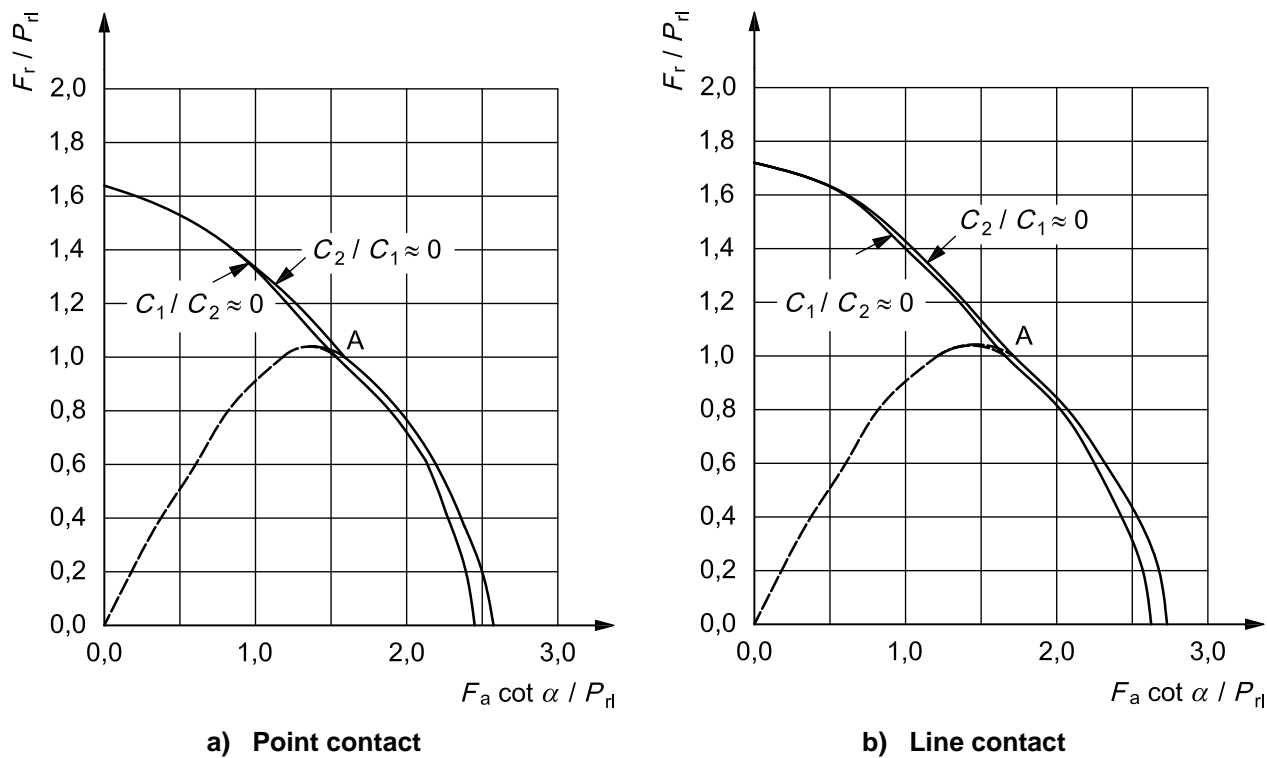
$$\left(\frac{\cos \alpha}{\cos \alpha'} - 1 \right)^{3/2} \sin \alpha' = \left[\frac{c_c}{(2r/D_w) - 1} \right]^{3/2} \frac{F_a}{Z D_w^2} \quad (66)$$

where

c_c is a compression constant, which depends on the modulus of elasticity and the conformity $2r/D_w$;

r is a cross-sectional raceway groove radius;

D_w is the ball diameter.



Key

- A points A
- C_1 basic dynamic radial load rating of a rotating ring
- C_2 basic dynamic radial load rating of a stationary ring
- F_a axial load
- F_r radial load
- P_{rl} dynamic equivalent radial load for the loaded row considered as a single row bearing
- α nominal contact angle

Figure 2 — Dynamic equivalent radial load, P_{rl} , for double row bearings with constant contact angle, α

5.1.3 Theoretical dynamic equivalent radial load, P_r , for radial contact groove ball bearings

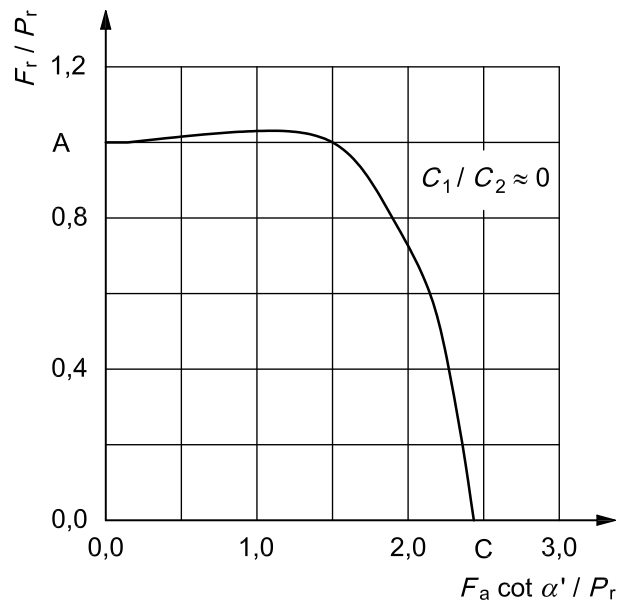
Figure 3 is applicable to radial contact groove ball bearings. The curve AC has been determined from Equation (54) and the approximate equation

$$\tan \alpha' \approx \left[\frac{2c}{(2r/D_w) - 1} \right]^{3/8} \left(1 - \frac{1}{2\varepsilon} \right)^{3/8} \left(\frac{F_a}{J_a i Z D_w^2} \right)^{1/4} \quad (67)$$

and gives the functional relationship between F_r/P_r and $F_a \cot \alpha'/P_r$ where α' is the contact angle calculated from Equation (68) (Reference [1])

$$\tan \alpha' \approx \left[\frac{2c}{(2r/D_w) - 1} \right]^{3/8} \left(\frac{F_a}{i Z D_w^2} \right)^{1/4} \quad (68)$$

Equation (68) is obtained from Equation (67) for a centric axial load $F_a = F_r = 0$, i.e. $\varepsilon = \infty$ and $J_a = 1$.



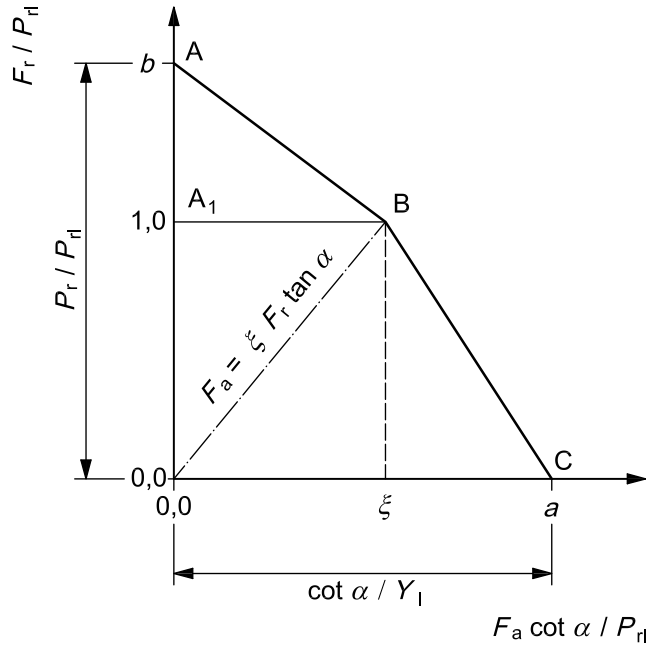
Key

- A point A
- C point C
- C_1 basic dynamic radial load rating of a rotating ring
- C_2 basic dynamic radial load rating of a stationary ring
- F_a axial load
- F_r radial load
- P_r dynamic equivalent radial load for radial bearing
- α' contact angle calculated from Equation (68)

Figure 3 — Dynamic equivalent radial load, P_r , for radial contact groove ball bearings

5.1.4 Practical expressions for dynamic equivalent radial load, P_{rl} , for radial bearings with constant contact angle

From a practical standpoint, it is preferable to replace the theoretical curves in Figures 1 and 2 by broken lines A_1BC for single row bearings and ABC for double row bearings, as in Figure 4.



Key

- A, A₁, B, C points
- a intercept of line BC on abscissa (x-co-ordinate of point C)
- b intercept of line AB on ordinate (y-co-ordinate of point A)
- F_a axial load
- F_r radial load
- P_{rl} dynamic equivalent radial load for radial bearing, row I
- Y₁ axial load factor for radial bearing, row I
- α nominal contact angle
- ξ value of F_a cot α/P_{rl} at the x-co-ordinate of point B

Figure 4 — Dynamic equivalent radial load, P_{rl} , for radial bearings with constant contact angle, α

The equation for the straight line A_1B in Figure 4 is

$$\frac{F_r}{P_{rl}} = 1$$

Therefore, for $F_a/F_r \leq \xi \tan \alpha$, we have

$$P_{rl} = F_r \tag{69}$$

and the straight line passing through the points B ($\xi, 1$) and C ($a, 0$) is given by

$$\frac{(F_r/P_{rl}) - 1}{(F_a \cot \alpha / P_{rl}) - \xi} = \frac{-1}{a - \xi}$$

From this equation, for $F_a/F_r > \xi \tan \alpha$, it follows that

$$\left. \begin{aligned}
 P_{rl} &= \left(1 - \frac{\xi}{a}\right) F_r + \frac{1}{a} \cot \alpha F_a \equiv X_1 F_r + Y_1 F_a \\
 \text{where} \\
 X_1 &= 1 - \frac{\xi}{a} = 1 - \xi Y_1 \tan \alpha \\
 \text{Therefore, from Equation (59)} \\
 X_1 &= 1 - \frac{J_r(0,5)}{\sqrt{J_1(0,5) J_2(0,5)}} \frac{\xi}{\eta} \\
 Y_1 &= \frac{J_r(0,5) \cot \alpha}{\sqrt{J_1(0,5) J_2(0,5)}} \frac{1}{\eta}
 \end{aligned} \right\} \quad (70)$$

For the double row bearings, the equation for the straight line AB is

$$\frac{(F_r/P_{rl}) - b}{F_a \cot \alpha / P_{rl}} = \frac{1-b}{\xi}$$

From this,

$$P_{rl} = \frac{F_r}{b} + \left(\frac{b-1}{b}\right) \frac{F_a \cot \alpha}{\xi}$$

Therefore, for $F_a/F_r \leq \xi \tan \alpha$, it follows that

$$\left. \begin{aligned}
 P_r &= 2^{1-(1/w)} P_{rl} = F_r + \left[2^{1-(1/w)} - 1\right] \frac{\cot \alpha}{\xi} F_a \equiv X_3 F_r + Y_3 F_a \\
 \text{where} \\
 X_3 &= 1; Y_3 = \left[2^{1-(1/w)} - 1\right] \frac{1}{\xi} \cot \alpha
 \end{aligned} \right\} \quad (71)$$

Further, from Equation (70), which represents straight line BC, we find for $F_a/F_r > \xi \tan \alpha$

$$\left. \begin{aligned}
 P_r &= 2^{1-(1/w)} P_{rl} = 2^{1-(1/w)} X_1 F_r + 2^{1-(1/w)} Y_1 F_a \equiv X_2 F_r + Y_2 F_a \\
 \text{where} \\
 X_2 &= 2^{1-(1/w)} X_1; Y_2 = 2^{1-(1/w)} Y_1
 \end{aligned} \right\} \quad (72)$$

Integrating the above, Table 4 shows expressions of dynamic equivalent radial load, P_r , and of factors X and Y , for radial bearings with constant contact angle, α .

Table 4 — Expressions for dynamic equivalent radial load, P_r , and factors X and Y for radial bearings with constant contact angle, α

		Single row bearings	Double row bearings
Expressions	$\frac{F_a}{F_r} \leq e$	$P_r = F_r$	$P_r = X_3 F_r + Y_3 F_a$
	$\frac{F_a}{F_r} > e$	$P_r = X_1 F_r + Y_1 F_a$	$P_r = X_2 F_r + Y_2 F_a$
Radial load factor, X Axial load factor, Y		$X_1 = 1 - \frac{J_r(0,5)}{\sqrt{J_1(0,5) J_2(0,5)}} \frac{\xi}{\eta}$ $Y_1 = \frac{J_r(0,5) \cot \alpha}{\sqrt{J_1(0,5) J_2(0,5)}} \frac{1}{\eta}$	$\frac{X_2}{X_1} = \frac{Y_2}{Y_1} = 2^{1-(1/w)}$ $X_3 = 1$ $Y_3 = \frac{1}{\xi} [2^{1-(1/w)} - 1] \cot \alpha$
Life scatter measure, e		$e = \xi \tan \alpha$	

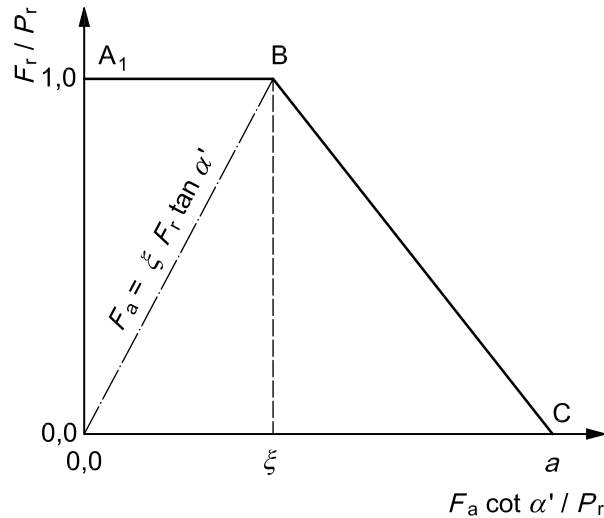
5.1.5 Practical expressions for dynamic equivalent radial load, P_r , for radial ball bearings

Generally, the contact angle of radial ball bearings varies with the load, but Table 4 can be approximately applicable to angular contact groove ball bearings, if α is replaced by contact angle α' under the axial load F_a given by Equation (66).

Therefore, according to Table 3,

$$\left. \begin{aligned} X_1 &= 1 - 0,4 \frac{\xi}{\eta}; & Y_1 &= \frac{0,4}{\eta} \cot \alpha' \\ X_2 &= 1,625 X_1; & Y_2 &= 1,625 Y_1 \\ X_3 &= 1; & Y_3 &= \frac{0,625}{\xi} \cot \alpha' \end{aligned} \right\} \quad (73)$$

For single row and double row radial contact groove ball bearings, the theoretical curve in Figure 3 is replaced by the broken line A_1BC in Figure 5.



Key

- A_1, B, C points
- a intercept of line BC on abscissa (x -co-ordinate of point C)
- F_a axial load
- F_r radial load
- P_r dynamic equivalent radial load for radial bearing
- α' contact angle calculated from Equation (68)
- ξ value of $F_a \cot \alpha' / P_r$ at point B (and its x -co-ordinate)

Figure 5 — Dynamic equivalent radial load, P_r , for radial contact groove ball bearings

For this type of bearing,

$$\left. \begin{aligned} X_1 = X_2 = 1 - 0,4 \frac{\xi}{\eta} \\ Y_1 = Y_2 = 0,4 \frac{\cot \alpha'}{\eta} \\ X_3 = 1; Y_3 = 0 \end{aligned} \right\} \quad (74)$$

For self-aligning ball bearings, the contact angle can be considered as independent of the load ($\alpha' = \alpha$); also η can be assumed to be unity.

5.1.6 Practical expressions for dynamic equivalent axial load, P_a , for thrust bearings

The radial and axial load factors, X_a and Y_a , for single and double direction bearings with $\alpha \neq 90^\circ$ are obtained on the basis of the expressions for dynamic equivalent radial load, P_r , for single row and double row radial bearings, respectively.

That is, for single direction bearings, when $F_a / F_r > \xi \tan \alpha$

$$Y_1 P_a = P_r = X_1 F_r + Y_1 F_a$$

So

$$\left. \begin{aligned} P_a &= \frac{X_1}{Y_1} F_r + F_a \equiv X_{a1} F_r + Y_{a1} F_a \\ \text{where} \\ X_{a1} &= \frac{X_1}{Y_1}; Y_{a1} = 1 \end{aligned} \right\} \quad (75)$$

and for double direction bearings, when $F_a/F_r > \zeta \tan \alpha$,

$$\left. \begin{aligned} P_a &= \frac{X_2}{Y_2} F_r + F_a \equiv X_{a2} F_r + Y_{a2} F_a \\ \text{where} \\ X_{a2} &= \frac{X_2}{Y_2}; Y_{a2} = 1 \end{aligned} \right\} \quad (76)$$

Further, when $F_a/F_r \leq \zeta \tan \alpha$, approximately

$$Y_2 P_a = P_r = X_3 F_r + Y_3 F_a$$

therefore

$$\left. \begin{aligned} P_a &= \frac{X_3}{Y_2} F_r + \frac{Y_3}{Y_2} F_a \equiv X_{a3} F_r + Y_{a3} F_a \\ \text{where} \\ X_{a3} &= \frac{X_3}{Y_2}; Y_{a3} = \frac{Y_3}{Y_2} \end{aligned} \right\} \quad (77)$$

Integrating the above, Table 5 shows expressions for dynamic equivalent axial load, P_a , for thrust bearings and factors X_a and Y_a .

Table 5 — Expressions for dynamic equivalent axial load, P_a , and factors X_a and Y_a for thrust bearings

		Single direction bearings	Double direction bearings
Expressions	$\frac{F_a}{F_r} \leq e$	—	$P_a = X_{a3} F_r + Y_{a3} F_a$
	$\frac{F_a}{F_r} > e$	$P_a = X_{a1} F_r + Y_{a1} F_a$	$P_a = X_{a2} F_r + Y_{a2} F_a$
Radial load factor, X_a Axial load factor, Y_a		$X_{a1} = \frac{X_1}{Y_1}$ $Y_{a1} = 1$	$X_{a2} = \frac{X_2}{Y_2}$ $Y_{a2} = 1$ $X_{a3} = \frac{X_3}{Y_2}$ $Y_{a3} = \frac{Y_3}{Y_2}$
Life scatter measure, e		$e = \zeta \tan \alpha$	

5.2 Factors X , Y , and e

5.2.1 Radial ball bearings

5.2.1.1 Values of ξ

For single row radial contact groove ball bearings, Reference [1] gives a value of $\xi = 1,2$ based on the results of tests, and for other bearings $\xi = 1,5$ which are close to the theoretical curves. However, based on later tests, ISO/R281 took values of $\xi = 1,05$ for radial contact groove ball bearings and single row angular contact groove types with $\alpha = 5^\circ$; $\xi = 1,25$ for other angular contact groove types; and $\xi = 1,5$ for self-aligning types (Reference [3]).

5.2.1.2 Values of η

The reduction factor, η , depends on the contact angle, α , and is given by

$$\eta = 1 - k \sin \alpha \quad (78)$$

Based on experience and preliminary tests, Reference [1] gives $k = 0,4$ and Reference [2] $k = 0,15$ to $0,33$. In ISO/R281, $k = 0,4$ ($= 1/2,5$) was used for radial contact groove bearings ($\alpha = 5^\circ$) and angular contact groove bearings with $\alpha = 5^\circ$, 10° and 15° and $k = (1/2,75)$ is used for angular contact groove bearings with $\alpha = 20^\circ$ to 45° (Reference [3]).

NOTE ISO/R281 did not include factors for bearings with $\alpha = 45^\circ$. Factors for this angle are specified in ISO 281:2007.

5.2.1.3 Values of contact angle α'

For radial contact groove ball bearings as well as angular contact groove bearings with nominal contact angle $\alpha \leq 15^\circ$, the real contact angle varies considerably with the load. Consequently, ISO 281:2007, Table 3, gives all factors as functions of the relative axial load.

The values of contact angle, α' , under an axial load, F_a , can be calculated from

$$\left(\frac{\cos 5^\circ}{\cos \alpha'} - 1 \right)^{3/2} \sin \alpha' = \left[\frac{c}{(2r/D_w) - 1} \right]^{3/2} \frac{F_a}{i Z D_w^2} \quad (79)$$

for radial contact groove ball bearings (considering them as angular contact groove bearings with a nominal contact angle, $\alpha = 5^\circ$), and from Equation (66) for angular contact groove bearings with a nominal contact angle, α .

For $2r/D_w = 1,035$, $c = 0,000\,438\,71$ is given, with units in newtons and millimetres.

Table 6 shows the values of contact angle α' calculated from Equations (66) and (79) for $2r/D_w = 1,035$.

For angular contact groove ball bearings with $\alpha \geq 20^\circ$, the influence of the axial load on the contact angle is comparatively small and therefore ISO 281:2007, Table 5, has only one set of X , Y , and e factors for each α . With regard to the calculation rules applied to these bearings, see 5.2.2.3.

**Table 6 — Values of contact angle α' for radial and angular contact groove ball bearings
($\alpha = 5^\circ, 10^\circ$ and 15°)**

$F_a/Z D_w^2$ ^a		$\alpha = 5^\circ$	$\alpha = 10^\circ$	$\alpha = 15^\circ$
lbf/in ²	MPa ^b	α'		
25	0,172 37	10,230°	12,953°	16,781°
50	0,344 74	11,811°	14,177°	17,652°
100	0,689 48	13,734°	15,768°	18,866°
150	1,034 21	15,037°	16,893°	19,767°
200	1,378 95	16,048°	17,786°	20,503°
300	2,068 4	17,607°	19,187°	21,688°
500	3,447 4	19,809°	21,207°	23,488°
750	5,171 1	21,761°	23,028°	25,075°
1 000	6,894 8	23,263°	24,444°	26,360°

^a For radial contact groove bearings $F_a/i Z D_w^2$
^b 1 MPa = 1 N/mm²

5.2.2 Values of X , Y , and e for each type of radial ball bearing

Integrating the above, methods of calculating values of X , Y , and e are as follows (see Tables 10 and 11).

5.2.2.1 Radial contact groove ball bearings

$$X_1 = X_2 = 1 - \frac{0,4 \times 1,05}{1 - 0,4 \sin 5^\circ} = 0,564 8 \approx 0,56$$

$$Y_1 = Y_2 = \frac{0,4 \cot \alpha'}{1 - 0,4 \sin 5^\circ} = 0,414 45 \cot \alpha'$$

$$e = 1,05 \tan \alpha'$$

The calculated Y_1 value of $0,964 1 \approx 0,96$ for $F_a/i Z D_w^2 = 6,89$ MPa is adjusted to 1,00 in consideration of the relationship with the value of Y_1 for angular contact groove type with $\alpha \geq 20^\circ$ (see Figure 6); namely, the calculated contact angle, α' , of $23,262^\circ$ is adjusted to $22,512^\circ$ ($\alpha' = \tan^{-1} 0,414 45$). Therefore, the calculated e value of $0,451 42 \approx 0,45$ becomes $0,435 2 \approx 0,44$ [$e = 1,05 \tan 22,512^\circ$ or $0,4 \times 1,05 / (1 - 0,4 \sin 5^\circ)$].

5.2.2.2 Angular contact groove ball bearings with $\alpha \leq 15^\circ$

For single row bearings with $\alpha = 5^\circ$, the values of X_1 , Y_1 , and e are the same as those for the radial contact groove type above.

For double row bearings with $\alpha = 5^\circ$

$$X_2 = 1,625 \times 0,48 = 0,78$$

because

$$X_1 = 1 - \frac{0,4 \times 1,25}{1 - 0,4 \sin 5^\circ} = 0,4819 \approx 0,48$$

$$Y_2 = 1,625 Y_1$$

where

$$Y_1 = \frac{0,4 \cot \alpha'}{1 - 0,4 \sin 5^\circ} = 0,41445 \cot \alpha'$$

$$Y_3 = \frac{0,625 \cot \alpha'}{1,25} = 0,5 \cot \alpha'$$

$$e = 1,25 \tan \alpha'$$

For $F_a / ZD_W^2 = 6,89$ MPa, the contact angle, α' , of $22,512^\circ$ is used. Therefore,

$$Y_2 = 1,625 \times 1 = 1,625 \approx 1,63$$

$$Y_3 = 0,5 \cot 22,512^\circ$$

or

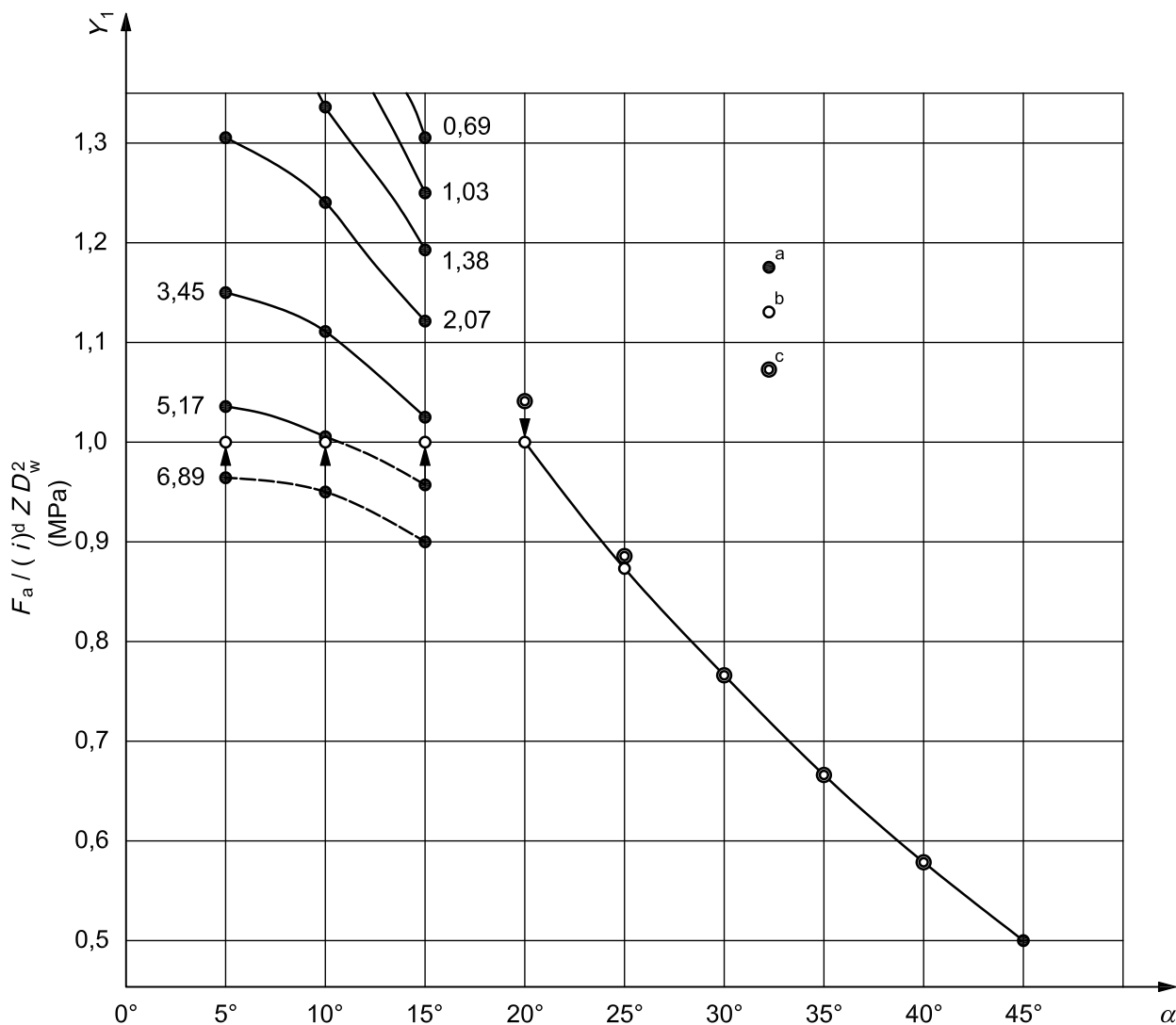
$$Y_3 = 1,5625 (1 - 0,4 \sin 5^\circ) = 1,2064 \approx 1,21$$

and

$$e = 1,25 \tan 22,512^\circ$$

or

$$e = \frac{0,4 \times 1,25}{1 - 0,4 \sin 5^\circ} = 0,5181 \approx 0,52$$



Key

- D_w ball diameter
- F_a axial load
- i number of rows of balls or rollers
- Y_1 axial load factor
- Z number of balls or rollers per row
- α nominal contact angle
- a Calculated values.
- b Adjusted values.
- c Values given in TC 4 N36.
- d Include factor i for radial contact groove bearings.

Figure 6 — Adjustment of Y_1 values for radial and angular contact groove ball bearings

For bearings with $\alpha = 10^\circ$ and $\alpha = 15^\circ$,

$$X_1 = 1 - \frac{0,4 \times 1,25}{1 - 0,4 \sin \alpha}$$

$$X_2 = 1,625 X_1$$

$$Y_1 = \frac{0,4 \cot \alpha'}{1 - 0,4 \sin \alpha}$$

$$Y_2 = 1,625 Y_1$$

$$Y_3 = \frac{0,625}{1,25} \cot \alpha' = 0,5 \cot \alpha'$$

$$e = 1,25 \tan \alpha'$$

Namely, for $\alpha = 10^\circ$, $X_1 = 0,462\ 7 \approx 0,46$, $Y_1 = 0,429\ 86 \cot \alpha'$, and for $\alpha = 15^\circ$, $X_1 = 0,442\ 3 \approx 0,44$, $Y_1 = 0,446\ 19 \cot \alpha'$.

For the above-stated reason, in the case of the calculated value of Y_1 being less than 1, Y_1 has to be set equal to 1,00 (see Figure 6). Therefore, we have for $F_a/i Z D_W^2 = 6,89$ MPa

$$Y_2 = 1,625 \approx 1,63$$

$$Y_3 = 0,5 \cot 23,261^\circ$$

or

$$Y_3 = 1,25 (1 - 0,4 \sin 10^\circ) = 1,632\ 2 \approx 1,16$$

and

$$e = 1,25 \tan 23,261^\circ$$

or

$$e = \frac{0,4 \times 1,25}{1 - 0,4 \sin 10^\circ} = 0,537\ 3 \approx 0,54$$

and also for $F_a/ZD_W^2 = 5,17$ MPa and $F_a/ZD_W^2 = 6,89$ MPa

$$Y_2 = 1,625 \times 1 = 1,625 \approx 1,63$$

$$Y_3 = 0,5 \cot 24,046^\circ$$

or

$$Y_3 = 1,25 (1 - 0,4 \sin 15^\circ) = 1,120\ 6 \approx 1,12$$

and

$$e = 1,25 \tan 24,046^\circ$$

or

$$e = \frac{0,4 \times 1,25}{1 - 0,4 \sin 15^\circ} = 0,577\ 7 \approx 0,56$$

5.2.2.3 Angular contact groove ball bearings with $\alpha = 20^\circ$ to $\alpha = 45^\circ$

$$X_1 = 1 - \frac{0,4 \times 1,25}{1 - (1/2,75) \sin \alpha}$$

See Table 7.

$$X_2 = 1,625 X_1$$

Table 7 — Values of X_1 for bearings with $\alpha = 20^\circ$ to $\alpha = 45^\circ$

α	X_1
20°	0,429 0 \approx 0,43
25°	0,409 2 \approx 0,41
30°	0,388 9 \approx 0,39
35°	0,368 2 \approx 0,37
40°	0,347 5 \approx 0,35
45°	0,326 9 \approx 0,33

For values of Y_1 , in principle, the values in Table 8, taken from document ISO/TC 4 N36 (= TC 4 N56 = TC 4 N110), are used (see Note), where the first and second values are adjusted, in consideration of the relationship with the values of Y_1 for $\alpha \leq 15^\circ$ (see Figure 6).

Table 8 — Values of Y_1

α	Y_1
20°	1,04 adjusted to 1,00
25°	0,89 adjusted to 0,87
30°	0,76
35°	0,66
40°	0,57
(45°)	(0,50)

Then values of Y_2 , e , and Y_3 are calculated from Equations (80) which are obtained from Equations (73):

$$\left. \begin{aligned} Y_2 &= 1,625 Y_1 \\ e &= \frac{1 - X_1}{Y_1} \\ Y_3 &= \frac{0,625}{e} \end{aligned} \right\} \quad (80)$$

NOTE The values of Y_1 are obtained from Equation (81):

$$Y_1 = \frac{0,4}{\eta} \cot \alpha' = \frac{0,4}{1 - (1/3) \sin \alpha'} \cot \alpha' \quad (81)$$

where the values of contact angle α' are determined by the equation

$$\cos \alpha' = \cos \alpha \, 0,972 \, 402$$

as in Table 9:

Table 9 — Values of α'

α	α'
20°	23,97°
25°	28,20°
30°	32,63°
35°	37,20°
40°	41,85°
(45°)	(46,56°)

The relation between α and α' is obtained from Equation (66) for

$$\frac{2r}{D_w} = \frac{r_i}{D_w} + \frac{r_e}{D_w} = 0,5175 + 0,53 = 1,0475$$

$$c_c = 0,00045835$$

and the rolling body load, in megapascals, is given by

$$\frac{F_a}{Z D_w^2 \sin \alpha'} = 4,9033$$

This is equivalent to 0,5 kgf/mm².

Moreover, for bearings with $\alpha = 45^\circ$, which were not included in ISO/R281, a Y_1 value of 0,4986 \approx 0,50 is determined from the same Equation (81), and is specified in ISO 281-1:1977 together with the values of remaining factors Y_2 , Y_3 , and e , which are obtained from the Equations (80).

5.2.2.4 Self-aligning ball bearings

Taking $\alpha' = \alpha$, $\eta = 1$, and $\xi = 1,5$,

$$X_1 = 1 - 0,4 \times 1,5 = 0,4$$

$$X_2 = 1,625 \times 0,4 = 0,65$$

$$Y_1 = 0,4 \cot \alpha$$

$$Y_2 = 1,625 Y_1 = 0,65 \cot \alpha$$

$$Y_3 = \frac{0,625}{1,5} \cot \alpha = 0,4167 \cot \alpha \approx 0,42 \cot \alpha$$

$$e = 1,5 \tan \alpha$$

5.2.3 Tabulation of factors X , Y , and e for radial ball bearings

Table 10 summarizes the basic equations for calculating the factors X , Y , and e as well as α' , ξ , and η values for each type of radial ball bearing.

Table 10 — Summary of factors X , Y and e for radial ball bearings

Bearing type	Single row bearings			Double row bearings			e	α'	ξ	η
	$\frac{F_a}{F_r} > e$	$\frac{F_a}{F_r} \leq e$	$\frac{F_a}{F_r} > e$	X_3	Y_3	X_2				
Radial contact groove bearings	X_1	Y_1	X_2	Y_2	X_3	Y_3	X_2	Y_2		
		$0,4 \frac{\cot \alpha'}{\eta} \geq 1,00$	$1 - \frac{0,4\xi}{\eta}$	$0,4 \frac{\cot \alpha'}{\eta}$	0	$1 - \frac{0,4\xi}{\eta}$	$0,4 \frac{\cot \alpha'}{\eta} \geq 1,00$	$\xi \tan \alpha' \leq 0,4 \frac{\xi}{\eta}$	1,05	$1 - \frac{\sin 5^\circ}{2,5}$
		$1 - \frac{0,4\xi}{\eta}$			$0,625 \frac{\cot \alpha'}{\xi} \geq 1,5625 \frac{\eta}{\xi}$				1,05 (for single row) 1,25 (for double row)	$1 - \frac{\sin \alpha}{2,5}$
Angular contact groove bearings										
Self-aligning bearings										
a	Equation (1) $\frac{\cos 5^\circ}{\cos \alpha'} = 1 + 0,012 534 \left(\frac{F_a}{i Z D_w^2 \sin \alpha'} \right)^{2/3}$									
b	Equation (2) $\frac{\cos \alpha^\circ}{\cos \alpha'} = 1 + 0,012 534 \left(\frac{F_a}{i Z D_w^2 \sin \alpha'} \right)^{2/3}$									
c	Determined by the equation $Y_1 = 0,4 \cot \alpha' / [1 - (1/3) \sin \alpha']$, where α' is determined from the equation $\cos \alpha' = \cos \alpha 0,972 4$.									
NOTE	The Y_1 values 1,04 and 0,89 obtained for $\alpha = 20^\circ$ and $\alpha = 25^\circ$ are adjusted to 1,00 and 0,87, respectively, to fit the data for $\alpha < 20^\circ$ smoothly.									

5.2.4 Calculated values Y and e different from standard

Table 11 shows the Y and e values calculated using values of contact angle α' given in Table 6, and which differ from the values given in ISO 281:2007, Table 2. The maximum discrepancy is within $\pm 0,02$.

This slight difference is due to the values of factors Y and e being related to contact angle α' . However, α' cannot be calculated directly for the given values of F_a/ZD_w^2 or F_a/iZD_w^2 from Equations (66) and (79).

Therefore, the discrepancies are thought to arise from the inaccuracy of the calculated values of contact angles α' .

Table 11 — Calculated values of Y and e different from ISO 281:2007, Table 2 ($\alpha \leq 15^\circ$)

F_a / ZD_w^2 ^a MPa		0,172	0,345	0,689	1,03	1,38	2,07	3,45	5,17	6,89	
Radial contact groove bearings	Y_1		1,98	1,70	1,54	1,44				b	
	e						0,33			b	
Angular contact groove bearings	$\alpha = 5^\circ$, double row bearings	Y_2		3,22	2,76	2,50	2,34				b
		Y_3	2,77	2,39	2,05	1,86	1,74			1,25	b
		e			0,31						b
	$\alpha = 10^\circ$	Y_1	1,87	1,70		1,42		1,24	1,11		b
		Y_2	3,04	2,76		2,31		2,02	1,80		b
		Y_3	2,17		1,77	1,65	1,56	1,44	1,29	1,18	b
		e			0,35			0,43	0,48	0,53	b
	$\alpha = 15^\circ$	Y_1	1,48		1,31	1,24			1,03	b	b
		Y_2	2,41		2,13	2,02			1,67	b	b
		Y_3	1,66			1,39			1,15	b	b
		e				0,45			0,54	b	b

^a For radial contact groove bearings, F_a/iZD_w^2 .

^b Adjusted values.

5.2.5 Thrust ball bearings

5.2.5.1 Fundamental equations

From Tables 3, 4, and 5, fundamental equations that determine factors X , Y , and e are obtained, as shown in Table 12.

Table 12 — Fundamental equations for factors X , Y , and e

Factor	Fundamental equation
$X_{a1} = X_{a2}$	$(2,5 \eta - \xi) \tan \alpha$
X_{a3}	$\frac{20}{13} \eta \tan \alpha$
$Y_{a1} = Y_{a2}$	1
Y_{a3}	$\frac{25 \eta}{26 \xi}$
e	$\xi \tan \alpha$

5.2.5.2 Values of ξ and η

Variable ξ is taken to have the same value of 1,25 as for angular contact groove ball bearings, and η is taken as $1 - (1/3)\sin \alpha$.

5.2.5.3 Values of contact angle

As variation of contact angle with the axial load, F_a , is very small in thrust ball bearings with large contact angle, the nominal contact angle α can be used.

5.2.5.4 Values of X_a , Y_a , and e

Integrating the above, values of X_a , Y_a , and e are calculated as follows:

$$\left. \begin{aligned} X_{a1} = X_{a2} &= \left[2,5 \left(1 - \frac{1}{3} \sin \alpha \right) - 1,25 \right] \tan \alpha \\ &= 1,25 \left(1 - \frac{2}{3} \sin \alpha \right) \tan \alpha \\ X_{a3} &= \frac{20}{13} \left(1 - \frac{1}{3} \sin \alpha \right) \tan \alpha \\ Y_{a1} = Y_{a2} &= 1 \\ Y_{a3} &= \frac{25}{26} \left(1 - \frac{1}{3} \sin \alpha \right) \frac{1}{1,25} = \frac{10}{13} \left(1 - \frac{1}{3} \sin \alpha \right) \\ e &= 1,25 \tan \alpha \end{aligned} \right\} \quad (82)$$

The values of factors X , Y , and e for $\alpha = 45^\circ$ to $\alpha = 85^\circ$ in ISO 281:2007, Table 5, are calculated from Equation (82). ISO/R281 prescribed the values only for $\alpha = 45^\circ$, $\alpha = 60^\circ$, and $\alpha = 75^\circ$; a Y_{a3} value of 0,54 for $\alpha = 60^\circ$ was found to be incorrect and the amended value of 0,55 appears in ISO 281:2007.

5.2.6 Radial roller bearings

5.2.6.1 Values of ξ and η

For radial roller bearings, $\xi = 1,5$ and $\eta = 1 - 0,15 \sin \alpha$ are used (Reference [2]).

5.2.6.2 Values of X , Y , and e

For radial roller bearings, three different types of contact are distinguished between the rollers and bearing rings, namely:

- point contact against both rings;
- line contact against both rings; and
- line contact against one ring and point contact against the other.

Table 13 shows the values of factors X , Y , and e for the three different cases of contacts and for contact angles $\alpha = 0^\circ$, $\alpha = 20^\circ$, and $\alpha = 40^\circ$. They are calculated from Tables 3 and 4.

Table 13 — Calculated values of X , Y , and e for bearings with a) point contact, b) line contact and c) line and points contacts

α	Bearing contact type	X_1	$\frac{Y_1}{\cot \alpha}$	X_3	$\frac{Y_3}{\cot \alpha}$	X_2	$\frac{Y_2}{\cot \alpha}$	$\frac{e}{\tan \alpha}$
0°	a)	0,40	0,40	1	0,42	0,65	0,65	1,5
	b)	0,44	0,37	1	0,48	0,75	0,63	1,5
	c)	0,42	0,38	1	0,45	0,70	0,63	1,5
20°	a)	0,37	0,42	1	0,42	0,60	0,68	1,5
	b)	0,41	0,39	1	0,48	0,70	0,67	1,5
	c)	0,39	0,40	1	0,45	0,65	0,67	1,5
40°	a)	0,34	0,44	1	0,42	0,55	0,72	1,5
	b)	0,38	0,41	1	0,48	0,65	0,70	1,5
	c)	0,36	0,42	1	0,45	0,60	0,70	1,5
Mean values		0,39	0,40	1	0,45	0,65	0,67	1,5
For practical use		0,4	0,4	1	0,45	0,67	0,67	1,5

The mean values of these factors are shown in the penultimate row of Table 13. For practical use, these values should be rounded off. They are listed under the mean values. Here, the value of X_2 has been changed from 0,65 to 0,67 to take account of the relation between Y_1 and Y_2 .

Therefore,

$$\left. \begin{aligned} X_1 &= 0,4 & Y_1 &= 0,4 \cot \alpha \\ X_2 &= 0,67 & Y_2 &= 0,67 \cot \alpha \\ X_3 &= 1 & Y_3 &= 0,45 \cot \alpha \\ e &= 1,5 \tan \alpha \end{aligned} \right\} \quad (83)$$

5.2.7 Thrust roller bearings

5.2.7.1 Values of ξ and η

As in the case of radial roller bearings, $\xi = 1,5$ and $\eta = 1 - 0,15 \sin \alpha$ are used.

5.2.7.2 Values of X_a , Y_a , and e

According to Table 5 and Equations (83).

$$\left. \begin{aligned} X_{a1} &= \tan \alpha & Y_{a1} &= 1 \\ X_{a2} &= \tan \alpha & Y_{a2} &= 1 \\ X_{a3} &= 1,4925 \tan \alpha \approx 1,5 \tan \alpha & Y_{a3} &= 0,6716 \approx 0,67 \\ e &= 1,5 \tan \alpha \end{aligned} \right\} \quad (84)$$

6 Basic rating life

The basic rating life of rolling bearings is the life associated with 90 % reliability for an individual rolling bearing, or a group of apparently identical rolling bearings operating under the same conditions.

Equations (85) and (86) are derived from Equations (4), (5), and (6):

$$QL_{10}^{3e/(c-h+2)} = Q_C \quad (\text{point contact}) \quad (85)$$

$$QL_{10}^{2e/(c-h+1)} = Q_C \quad (\text{line contact}) \quad (86)$$

Since the rolling element load is proportional to the bearing load, Q_C and Q are proportional to the basic dynamic load rating, C_r or C_a , and the bearing dynamic equivalent radial load, P_r , and dynamic equivalent axial load, P_a , respectively. From Equations (85) and (86), the following equations are found

$$\left. \begin{array}{l} L_{10} = \left(\frac{C_r}{P_r} \right)^{(c-h+2)/3e} \\ \text{or} \\ L_{10} = \left(\frac{C_a}{P_a} \right)^{(c-h+2)/3e} \end{array} \right\} \quad (\text{point contact}) \quad (87)$$

$$\left. \begin{array}{l} L_{10} = \left(\frac{C_r}{P_r} \right)^{(c-h+1)/2e} \\ \text{or} \\ L_{10} = \left(\frac{C_a}{P_a} \right)^{(c-h+1)/2e} \end{array} \right\} \quad (\text{line contact}) \quad (88)$$

Substituting experimental constants $e = 10/9$ (point contact), $e = 9/8$ (line contact), $c = 31/3$ and $h = 7/3$ into Equations (87) and (88), respectively,

$$\left. \begin{array}{l} L_{10} = \left(\frac{C_r}{P_r} \right)^3 \\ \text{or} \\ L_{10} = \left(\frac{C_a}{P_a} \right)^3 \end{array} \right\} \quad (\text{point contact}) \quad (89)$$

$$\left. \begin{array}{l} L_{10} = \left(\frac{C_r}{P_r} \right)^4 \\ \text{or} \\ L_{10} = \left(\frac{C_a}{P_a} \right)^4 \end{array} \right\} \quad (\text{line contact}) \quad (90)$$

Generally speaking, the contact between the rollers and the raceways transforms from a point to a line contact at a certain load so that the life exponent varies from 3 to 4 for different loading intervals within the same bearing. A uniform method of calculation is desired, however, applicable to all roller bearings and all loading intervals. In this regard, it is convenient to apply the same life equations to all types of roller bearings, namely:

$$\left. \begin{aligned} L_{10} &= \left(\frac{C_r}{P_r} \right)^{10/3} \\ \text{or} \\ L_{10} &= \left(\frac{C_a}{P_a} \right)^{10/3} \end{aligned} \right\} \quad (91)$$

Differences between actual and calculated life values, caused by the use of this single exponent, are reduced by the use of a compensatory adjustment of the load rating (see 4.4).

7 Life adjustment factor for reliability

A relationship between the bearing life and its probability of survival is expressed as Equation (92) which is derived from Correlation (1):

$$\ln \frac{1}{S} = AL^e \quad (92)$$

where

- S is the probability of survival;
- A is a constant of proportionality;
- L is the bearing life;
- e is the Weibull slope.

Inserting the basic rating life, L_{10} , as L for $S = 0,9$ into Equation (92) gives a constant of proportionality, A , as:

$$A = \frac{\ln(1/0,9)}{L_{10}^e} \quad (93)$$

From Equations (92) and (93), Equation (94) can be derived:

$$L_n = a_1 L_{10} \quad (94)$$

where a_1 is the life adjustment factor for reliability, given by:

$$a_1 = \left[\frac{\ln(1/S)}{\ln(1/0,9)} \right]^{1/e} \quad (95)$$

The failure distribution curve below 10 % failure probability is bent down towards higher fatigue lives (Reference [4]).

The failure distribution curve below 10 % of the failure probability has been approached by a Weibull distribution with the Weibull slope $e = 1,5$.

Values for the life modification factor for reliability, a_1 , shown in ISO 281:2007, Table 12, can be calculated from Equation (95) with $e = 1,5$.

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